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NANO Δ-GENERALIZED CONTINUOUS IN NANO TOPOLOGICAL SPACES

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Abstract. In this article, we introduced and studied $n\Delta$-open sets in nano topological spaces [**9**]. We offer a new class of sets called $n\Delta g$-closed sets in nano topological spaces and we study some of its basic properties [**4**]. We introduce $n\Delta$-continuous maps, $n\Delta g$-continuous maps, $n\Delta$-irresolute maps, $n\Delta g$-irresolute maps, contra $n\Delta$-continuous maps and contra $n\Delta g$-continuous maps in nano topological spaces and discuss some of their properties.

1. Introduction

Several notions of open-like and closed-like sets in nano topological spaces were introduced and studied. The beginning was with M. Lellis Thivagar and Carmel Richard who initiated the notion of nano forms of weakly open sets and nano continuity, [**5**, **6**]. The concept of nano continuity in nano topological spaces was extended to nano generalized continuity, [**1**]. However, another extension of continuity in nano topological spaces called nano generalized semi continuity was obtained in [**2**].

Recently, the idea of introducing new classes of nano continuity-like in nano topological spaces is still attracting many researchers, for example [**7**, **10**–**12**].

A set in a topological space is called Δ-open if it is the symmetric difference of two open sets. The notion of Δ-open sets appeared in [**8**] and in [**3**]. However, it was pointed out in [**8**] and in [**3**] that the notion of Δ-open sets is due to a preprint

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by M. Veera Kumar. The complement of a Δ-open set is Δ-closed.

A set in a nano topological space is called $n\Delta$-open if it is the symmetric difference of two nano open sets were initiated, [**9**].

Preliminary concepts required in our work are briefly recalled in section 2. In section 3, the concept of $n\Delta g$-continuous maps and $n\Delta g$-irresolute maps. We introduce contra $n\Delta g$-continuous map in nano topological spaces and discuss some of their properties.

2. Preliminaries

DEFINITION 2.1. *[**5**] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.*
 That is, $L_R(X) = \bigcup x_\epsilon U\{R(X) : R(X) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X.
(2) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.*
 That is, $U_R(X) = \bigcup x_\epsilon U\{R(X) : R(X) \cap X \neq \phi\}$
(3) *The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$.*
 That is, $B_R(X) = U_R(X) - L_R(X)$.

DEFINITION 2.2. *[**5**] If (U, $\tau_R(X)$) is the nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then*

(1) *The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by nint(A). That is, nint(A) is the largest nano open subset of A.*
(2) *The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by ncl(A). That is, ncl(A) is the smallest nano closed set containing A.*

DEFINITION 2.3. *[**8, 12**] A subset A of a space (X, τ) is called Δ-open if $A = (B - C) \cup (C - B)$, where B and C are open subsets of X. The complement of Δ-open sets is called Δ-closed sets.*

DEFINITION 2.4. *[**9**] A subset S of a space (U, $\tau_R(X)$) is said to be nano Δ-open set (in short, $n\Delta$-open) if $S = (A - B) \cup (B - A)$, where A and B* *are nano-open subsets in U. The complement of nano-Δ-open sets is called nano -Δ-closed sets.*

Definition 2.5. *[9] A subset S of a space (U, $\tau_R(X)$) is said to be semi nano Δ-open set (in short, snΔ-open) if S = (A − B) ∪ (B − A), where A and B are nano semi-open subsets in U. The complement of nano semi Δ-open sets is called nano semi Δ-closed sets.*

Definition 2.6. *[9] The nano interior of a set A is denoted by nano Δ-int(A) (briefly, nΔ-int(A)) and is defined as the union of all nΔ open sets contained in A. i.e., nΔ-int(A) = ∪ {G : G is nΔ-open and G ⊆ A }.*

Definition 2.7. *[9] The nano closure of a set A is denoted by nano Δ-cl(A) (briefly, nΔ-cl(A)) and is defined as the intersection of all nΔ-closed sets containing A. i.e., nΔ-cl(A) = ∩ {F : F is nΔ-closed and A ⊆ F}.*

Definition 2.8. *[4] A subset A of a space (U, $\tau_R(X)$) is called a nΔ-generalized closed (briefly, nΔg-closed) set if nΔcl(A) ⊆ T whenever A ⊆ T and T is nΔ-open in (U, $\tau_R(X)$).*
The complement of nΔg-closed set is called nΔg-open set.

Proposition 2.1. *[4] Every nΔ-closed set is nΔg-closed but not conversely.*

3. $n\Delta g$-continuous maps and $n\Delta g$-irresolute maps

Definition 3.1. *A map $f : (U, \tau_R(X)) \to (V, \tau_R(X)')$ is called nΔ-continuous (resp. nΔg-continuous) if $f^{-1}(G)$ is a nΔ-closed (resp. nΔg-closed) set of (U, $\tau_R(X)$) for every nΔ-closed set G of $(V, \tau_R(X)')$.*

Proposition 3.1. *Every nΔ-continuous is nΔg-continuous but not conversely.*

Proof. The proof follows from Proposition 2.1. □

Theorem 3.1. *If $f : (U, \tau_R(X)) \to (V, \tau_R(X)')$ is nΔg-continuous and $g : (V, \tau_R(X)') \to (W, \tau_R(X)'')$ is nΔ- continuous then $g \circ f : (U, \tau_R(X)) \to (W, \tau_R(X)'')$ is nΔg-continuous.*

Proof. Let K be $n\Delta$-closed set in W. Since g is $n\Delta$-continuous, g^{-1}(K) is $n\Delta$-closed in V. Since f is $n\Delta g$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(K)) is $n\Delta g$-closed in U. Therefore g ∘ f is $n\Delta g$-continuous. □

Proposition 3.2. *A map $f : (U, \tau_R(X)) \to (V, \tau_R(X)')$ is nΔg-continuous if and only if $f^{-1}(G)$ is nΔg-open in (U, $\tau_R(X)$) for every nΔ-open set G in (V, $\tau_R(X)'$) .*

Proof. Let f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) be $n\Delta g$-continuous and G be an $n\Delta$-open set in (V, $\tau_R(X)'$) . Then G^c is $n\Delta$-closed in (V, $\tau_R(X)'$) and since f is $n\Delta g$-continuous, $f^{-1}(G^c)$ is $n\Delta g$-closed in (U, $\tau_R(X)$). But $f^{-1}(G^c) = f^{-1}((G))^c$ and so f^{-1}(G) is $n\Delta g$-open in (U, $\tau_R(X)$).
Conversely, assume that f^{-1}(G) is $n\Delta g$-open in (U, $\tau_R(X)$) for each $n\Delta$-open set

G in (V, $\tau_R(X)'$) . Let F be a $n\Delta$-closed set in (V, $\tau_R(X)'$) . Then F^c is $n\Delta$-open in (V, $\tau_R(X)'$) and by assumption, $f^{-1}(F^c)$ is $n\Delta g$-open in (U, $\tau_R(X)$). Since $f^{-1}(F^c) = f^{-1}((F))^c$, we have $f^{-1}(F)$ is $n\Delta$-closed in (U, $\tau_R(X)$) and so f is $n\Delta g$-continuous. □

DEFINITION 3.2. *A map f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) is called*

1. *$n\Delta$-irresolute if f^{-1}(G) is a semi nano Δ-closed set of (U, $\tau_R(X)$) for every semi nano Δ-closed set G of (V, $\tau_R(X)'$) .*
2. *$n\Delta g$-irresolute if f^{-1}(G) is a $n\Delta g$-closed set of (U, $\tau_R(X)$) for every $n\Delta g$-closed set G of (V, $\tau_R(X)'$) .*

EXAMPLE 3.1. *Let U = {a, b, c} with U/ R= {{a}, {b, c}} and X = {a}. The nano topology $\tau_R(X)$ = {ϕ, {a}, U}. Then $n\Delta$-closed sets are ϕ, {a}, {b, c}, U and semi nano Δ-closed sets are ϕ, {a}, {b, c}, U. Let V = {a, b, c} with V/ R= {{a}, {b, c}} and X = {a, b}. The nano topology $\tau_R(X)'$ = {ϕ, {a}, {b, c}, V}. Then $n\Delta$-closed sets are ϕ, {a}, {b, c}, V and semi nano Δ-closed sets are ϕ, {a}, {b, c}, V. Define f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) be the identity map. Then f is $n\Delta$-irresolute.*

EXAMPLE 3.2. *Let U = {a, b, c} with U/ R= {{a}, {b, c}} and X = {a}. The nano topology $\tau_R(X)$ = {ϕ, {a}, U}. Then $n\Delta g$-closed sets are ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, U. Let V = {a, b, c} with V/ R= {{a}, {b, c}} and X = {a, b}. The nano topology $\tau_R(X)'$ = {ϕ, {a}, {b, c}, V}. Then $n\Delta g$-closed sets are ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, V. Define f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) be the identity map. Then f is $n\Delta g$-irresolute.*

THEOREM 3.2. *Every $n\Delta g$-irresolute map is $n\Delta g$-continuous but not conversely.*

PROOF. Let f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) be a $n\Delta g$-irresolute map. Let G be a $n\Delta$-closed set of (V, $\tau_R(X)'$) . Then by the Proposition 2.1, G is $n\Delta g$-closed. Since f is $n\Delta g$-irresolute, then f^{-1}(G) is a $n\Delta g$-closed set of (U, $\tau_R(X)$). Therefore f is $n\Delta g$-continuous. □

THEOREM 3.3. *Let f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) and g : (V, $\tau_R(X)'$) → (W, $\tau_R(X)''$) be any two maps. Then*

1. *$g \circ f$ is $n\Delta g$-continuous if g is $n\Delta$-continuous and f is $n\Delta g$-continuous.*
2. *$g \circ f$ is $n\Delta g$-irresolute if both f and g are $n\Delta g$-irresolute.*
3. *$g \circ f$ is $n\Delta g$-continuous if g is $n\Delta g$-continuous and f is $n\Delta g$-irresolute.*

PROOF. Omitted. □

DEFINITION 3.3. *A map f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) is called*

1. *contra $n\Delta$-continuous if f^{-1}(G) is a $n\Delta$-closed set of (U, $\tau_R(X)$) for every $n\Delta$-open set G of (V, $\tau_R(X)'$) .*
2. *contra $n\Delta g$-continuous if f^{-1}(G) is a $n\Delta g$-closed set of (U, $\tau_R(X)$) for every $n\Delta$-open set G of (V, $\tau_R(X)'$) .*

PROPOSITION 3.3. *Every contra $n\Delta$-continuous is contra $n\Delta g$-continuous but not conversely.*

PROOF. Let f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) be a contra $n\Delta$-continuous map and let G be any $n\Delta$-open set in (V, $\tau_R(X)'$) . Then, f^{-1}(G) is $n\Delta$-closed in U. Since every $n\Delta$-closed set is $n\Delta g$-closed, f^{-1}(G) is $n\Delta g$-closed in U. Therefore f is contra $n\Delta g$-continuous. □

EXAMPLE 3.3. *Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, b\}$. The nano topology $\tau_R(X) = \{\phi, \{a\}, \{b, c\}, U\}$. Then $n\Delta$-closed sets are ϕ, $\{a\}$, $\{b, c\}$, U and $n\Delta g$-closed sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, U. Let V = $\{a, b, c\}$ with $V/R = \{\{b\}, \{a, c\}, \{c, a\}\}$ and $X = \{a, c\}$. The nano topology $\tau_R(X)' = \{\phi, \{a, c\}, V\}$. Then $n\Delta$-open sets are ϕ, $\{b\}$, $\{a, c\}$, V. Define f : (U, $\tau_R(X)$) → (V, $\tau_R(X)'$) be the identity map. Then f is contra $n\Delta g$-continuous but not contra $n\Delta$-continuous, since $f^{-1}(\{a, c\}) = \{a, c\}$ is not $n\Delta$-closed in (U, $\tau_R(X)$).*

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