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RIGHT DERIVATIONS ON ORDERED SEMIRINGS

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Abstract. In this paper, we introduce the concept of right derivation on ordered semirings and we study some of the properties of right derivation on ordered semirings.

1. Introduction

The notion of a derivation was introduced long back, from the analytic theory is helpful to study the structures, properties of algebraic systems and has important role in characterizing of algebraic structures. The notion of derivation has also been generalized in various directions. A lot of research has been done on derivations and related. Over the last few decades several authors studied derivations in rings, semigroups, semirings and investigated the relationship between the commutativity of algebraic structures and the existence of specified derivations of algebraic structure. In 1990, Bresar and Vukman [**2**] established that a prime ring must be commutative if it admits a nonzero left derivation.Over the last few decades, several authors have investigated the relationship between the commutativity of ring R and the existence of certain specified derivations of R: The first result in this direction is due to Posner [**10**] in 1957. In the year Kim [**3**], [**4**] studied right derivation and generalized derivation of incline algebra. The notion of derivation of algebraic structures is useful for characterization of algebraic structures. The notion of derivation has also been generalized in various directions such as right derivation, left derivation, f-derivation, reverse derivation, orthogonal derivation, (f; g)-derivation, generalized right derivation, etc. Murali Krishna Rao and Venkateswarlu [**5**], [**6**] introduced the generalization on right derivation of Γ- incline and right derivation of ordered semirings. Semigroup, as the basic algebraic structure was used in the areas of

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theoretical computer science as well as in the solutions of graph theory, optimization theory and in particular for studying automata, coding theory and formal languages. The notion of a semiring was first introduced by Vandiver [**11**] in 1934 but semirings had appeared in studies on the theory of ideals of rings. A universal algebra $S = (S, +,)$ is called a semiring if and only if $(S, +), (S,)$ are semigroups which are connected by distributive laws, i:e., $a(b+c) = ab+ac, (a+b)c = ac+bc$, for all $a, b, c \in S$. A natural example of semiring is the set of all natural numbers under usual addition and multiplication of numbers. In particular, if I is the unit interval on the real line, then (I, max, min) is a semiring in which 0 is the additive identity and 1 is the multiplicative identity. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. In structure, semiring lies between semigroup and ring. Many semirings have order structure in addition to their algebraic structure. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring. Semirings are useful in the areas of theoretical computer science as well as in the solutions of graph theory and optimization theory in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches. In this paper, we introduced the concept of right derivation on ordered semirings and we study some of the properties of right derivation of ordered semirings.

2. Preliminaries

In this section we recall some important definitions on semirings, qrdered semirings that are necessary for this paper.

Definition 2.1. *A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and respectively) will be called a semiring provided*
(i) addition is a commutative operation (ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x0 = 0x = 0$ for all $x \in S$.

example 2.1. *Let M be the set of all natural numbers. Then (M, max, min) is a semiring.*

Definition 2.2. *Let M be a semiring. If there exists $1 \in M$ such that $a1 = 1a = a$, for all $a \in M$, is called an unity element of M then M is said to be semiring with unity.*

Definition 2.3. *An element a of a semiring S is called a regular element if there exists an element b of S such that $a = aba$.*

Definition 2.4. *A semiring S is called a regular semiring if every element of S is a regular element.*

Definition 2.5. *An element a of a semiring S is called a multiplicatively idempotent (an additively idempotent) element if $aa = a(a + a = a)$.*

Definition 2.6. *An element b of a semiring M is called an inverse element of a of M if $ab = ba = 1$.*

Definition 2.7. *A non-empty subset A of semiring M is called (i) a subsemiring of M if A is an additive subsemigroup of M and $AA \subseteq A$.*
(ii) a left(right) ideal of M if A is an additive subsemigroup of M and $MA \subseteq A(AM \subseteq A)$.
(iii) an ideal if A is an additive subsemigroup of $M, MA \subseteq A$ and $AM \subseteq A$.
(iv) a k-ideal if A is a subsemiring of $M, AM \subseteq A, MA \subseteq A$ and $x \in M, x + y \in A., y \in A$ then $x \in A$

Definition 2.8. *A semiring M is called a division semiring if for each non-zero element of M has multiplication inverse.*

Definition 2.9. *A semiring M is called an ordered semiring if it admits a compatible relation $\leqslant$. i.e. $\leqslant$ is a partial ordering on M satisfies the following conditions. If $a \leqslant b$ and $c \leqslant d$ then*
(i) $a + c \leqslant b + d, c + a \leqslant d + b$
(ii) $ac \leqslant bd$
(iii) $ca \leqslant db$, for all $a, b, c, d \in M$

Definition 2.10. *An ordered semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0x = x0 = 0$, for all $x \in M$.*

An ordered semiring M is said to be commutative semiring if $xy = yx$, for all $x, y \in M$

Definition 2.11. *A non zero element a in an ordered semiring M is said to be a zero divisor if there exists non zero element $b \in M$, such that $ab = ba = 0$.*

Definition 2.12. *An ordered semiring M with unity 1 and zero element 0 is called an integral ordered semiring if it has no zero divisors.*

Definition 2.13. *An ordered semiring M is said to be totally ordered semiring M if any two elements of M are comparable.*

Definition 2.14. *In an ordered semiring M*
(i) the semigroup $(M, +)$ is said to be positively ordered , if $a \leqslant a+b$ and $b \leqslant a+b$, for all $a, b \in M$.
(ii) the semigroup $(M, +)$ is said to be negatively ordered, if $a+b \leqslant a$ and $a+b \leqslant b$, for all $a, b \in M$.
(iii) the semigroup $(M,)$ is said to be positively ordered, if $a \leqslant ab$ and $b \leqslant a\alpha b$, for all $a, b \in M$.
(iv) the semigroup $(M,)$ is said to be negatively ordered if $ab \leqslant a$ and $ab \leqslant b$ for all $a, b \in M$.

Definition 2.15. *A non-empty subset A of an ordered semiring M is called a subsemiring M if $(A, +)$ is a subsemigroup of $(M, +)$ and $ab \in A$ for all $a, b \in A$.*

Definition 2.16. *Let M be an ordered semiring. A non-empty subset I of M is called a left (right) ideal of an ordered semiring M if I is closed under addition, $MI \subseteq I(IM \subseteq I)$ and if for any $a \in M, b \in I, a \leqslant b)a \in I.I$ is called an ideal of M if it is both a left ideal and a right ideal of M.*

Definition 2.17. *A non-empty subset A of ordered Γ-semiring M is called a k- ideal if A is an ideal and $x \in M, x+y \in A, y \in A$ then $x \in A$.*

Definition 2.18. *Let M and N be ordered semirings. A mapping $f : M \to N$ is called a homomorphism if*
(i) $f(a+b) = f(a) + f(b)$
(ii)$f(ab) = f(a)f(b)$, for all $a, b \in M. \in \Gamma$.

Definition 2.19. *Let M be an ordered semiring. A mapping $f : M \to M$ is called an endomorphism if*
(i)f is an onto,
(ii)$f(a+b) = f(a) + f(b)$,
(iii)$f(ab) = f(a)f(b)$, for all $a, b \in M$.

Definition 2.20. *Let M be an ordered semiring. A mapping $d : M \to M$ is called a derivation if it satisfies*
(i)$d(x+y) = d(x) + d(y)$
(ii)$d(xy) = d(x)y + xd(y)$ for all $x, y \in M$.

3. Right derivation of ordered semirings

Definition 3.1. *Let M be an ordered semiring. If a mapping $d : M \to M$ satisfies the following conditions*
(i)$d(x+y) = d(x) + d(y)$
(ii)$d(xy) = d(x)y + d(y)x$
(iii) If xy then $d(x) \leqslant d(y)$, for all $x, y \in M$. then d is called right derivation of M.

Theorem 3.1. *Let M be a commutative ordered semiring and $(M, +)$ is idempotent. For a fixed element $a \in M$, the mapping $d_a : M \to M$ given by $d_a(x) = xa$. for all $x \in M$, is a right derivation of M.*

Proof. Let M be a commutative ordered semiring and $a \in M$. Suppose $x, y \in M$.

$$\begin{aligned} d_a(x+y) &= (x+y)a \\ &= xa + ya \\ &= d_a(x) + d_a(y) \end{aligned}$$

and

$$\begin{aligned} d_a(xy) &= (xy)a \\ &= (xy)a + (xy)a \\ &= (xa)y + (ya)x = d_a(x)y + d_a(y)x. \end{aligned}$$

Suppose $x \leqslant y, xa \leqslant ya$ $d_a(x)(x) \leqslant d_a(y)$. Hence d_a is a right derivation of M $\square$

THEOREM 3.2. *Let d be a right derivation of an ordered semiring M. Then $d(0) = 0$.*

PROOF. Let d be a right derivation of ordered semiring M. Then

$$\begin{aligned} d(0) &= d(00) \\ &= d(0)0 + d(0)0 \\ &= 0 + 0 \\ &= 0. \end{aligned}$$

Therefore $d(0) = 0$ □

THEOREM 3.3. *Let d be a right derivation of an idempotent ordered semiring M and M is negatively ordered. Then $d(x) \leqslant x$, for all $x \in M$.*

PROOF. Let d be a right derivation of an idempotent ordered semiring M and M is negatively ordered. Then

$$\begin{aligned} d(x) &= d(xx) \\ &= d(x)x + d(x)x \\ &= d(x)x \\ \Rightarrow d(x) &= d(x)x \\ \Rightarrow d(x) &\leqslant x. \end{aligned}$$

□

THEOREM 3.4. *Let M be an ordered semiring and M be a negatively ordered. Then $d(xy) \leqslant d(x + y)$.*

PROOF. Let M be an ordered semiring and M be a negatively ordered. Suppose $x, y \in M$. Then $d(x)y \leqslant d(x)$ and $d(y)x \leqslant d(y)$. Therefore

$$\begin{aligned} d(xy) &= d(x)y + d(y)x \\ &= d(x) + d(y) \\ &= d(x + y). \end{aligned}$$

□

THEOREM 3.5. *Let M be an idempotent ordered semiring. If $d^2(x) = d(d(x)) = d(x)$ then $d(xd(x)) = d(x)$, for all $x \in M$.*

PROOF. Let M be an idempotent ordered semiring and $d^2(x) = d(d(x)) = d(x)$, for all $x \in M$. Then

$$\begin{aligned} d(xd(x)) &= d(x)d(x) + d(d(x))x \\ &= d(x) + d(x)x \\ &\leqslant d(x) + d(x) = d(x). \end{aligned}$$

Therefore $d(xd(x)) = d(x)$. □

THEOREM 3.6. *Let M be a commutative ordered semiring and d_1, d_2 be right derivation of M. Define $d_1d_2(x) = d_1(d_2(x))$, for all $x \in M$. If $d_1d_2 = 0$ then d_2d_1 is a right derivation of M.*

PROOF. Let M be a commutative ordered semiring and d_1, d_2 be right derivation of M. Define $d_1d_2(x) = d_1(d_2(x))$, for all $x \in M$. Suppose $d_1d_2 = 0$. Then $d_1d_2(xy) = 0$
$\Rightarrow d_2(x)d_1(y) + d_2(y)d_1(x) = 0 \cdots (1)$ and

$$\begin{aligned} d_2d_1(xy) &= d_2(d_1(xy)) \\ &= d_2(d_1(x)y + d_1(y)x) \\ &= d_2(d_1(x)y) + d_2(d_1(y)x) \\ &= d_2d_1(x)y + d_2(y)d_1(x) + d_2d_1(y)x) + d_2(x)d_1(y)(form(1). \end{aligned}$$

Obviously $d_2d_1(x + y) = d_2d_1(x) + d_2d_1(y)$. Hence d_2d_1 is a right derivation of commutative ordered semiring. □

DEFINITION 3.2. *Let M be an ordered semiring and d be a right derivation of M. An ideal I of ordered semiring M is called a d ideal if $d(I) = I$.*

EXAMPLE 3.1. *Zero ideal $\{0\}$ is a d ideal of ordered semiring. Since $d(0) = 0$. If d is onto derivation then $d(M) = M$. Hence M is a d ideal. Let d be a right derivation of ordered Γ semiring M. Define $kerd = \{x \in M | d(x) = 0\}$*

THEOREM 3.7. *Let d be a right derivation of idempotent non negatively ordered integral semiring M. Then $kerd$ is k ideal of ordered semiring M.*

PROOF. Let d be a right derivation of idempotent non negatively ordered integral semiring M. Suppose $x, y \in kerd$. Then $d(x) = 0$ and $d(y) = 0 \Rightarrow d(xy) = 0$. Therefore $xy \in kerd$ and $d(x + y) = 0$. Therefore $kerd$ is a subsemiring of M. Let $x \in M$ and $y \in kerd$ such that $x \leqslant y$. Then $d(y) = 0$.

$$\begin{aligned} \Rightarrow d(xy) &= d(x)y + d(y)x \\ &= d(x)y + 0x \\ &= d(x)y \end{aligned}$$

$$\begin{aligned} since x \leqslant y &\Rightarrow xy \leqslant yy \\ &\Rightarrow xy \leqslant 0 \\ &\Rightarrow d(xy) \leqslant d(0) \\ &\Rightarrow d(xy) \leqslant 0. \end{aligned}$$

Therefore $d(xy) = 0$.
Then $0 = d(x)y \Rightarrow d(x) - 0$ or $y = 0$.
If $d(x) = 0$ then $x \in kerd$. Suppose $y = 0$ and $x \leqslant y$. Then $x = 0 \Rightarrow x \in kerd$. Let $x + y \in kerd$ and $y \in kerd$. Then

$$\begin{aligned} & d(x + y) = d(y) = 0 \\ \Rightarrow\ & d(x) + d(y) = 0 \\ \Rightarrow\ & d(x) = 0 \end{aligned}$$

Therefore $x \in kerd$. Hence $kerd$ is a k ideal of M. □

THEOREM 3.8. *Let M be a non-negatively ordered semiring. Then $d(xy) \leqslant d(x)$, for all $x, y \in M$ and $d(xy) \leqslant d(y)$.*

PROOF. Let M be a non-negatively ordered semiring.

$$\begin{aligned} d(xy) &= d(x)y + d(y)x \\ &\leqslant d(x) + d(y) \\ &= d(x+y) \\ &\geqslant d(x) \end{aligned}$$

Similarly we can prove $d(xy) \leqslant d(y)$ □

The following proof of the routine verification

THEOREM 3.9. *Let M be an ordered semiring. If d_1, d_2 are right derivations of M and $d_1 + d_2(x) = d_1(x) + d_2(x)$, for all $x \in M$ then $d_1 + d_2$ is also a right derivation of M.*

EXAMPLE 3.2. *Let $M = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} \middle| a, b \in Q \right\}$ where Q is the set of all rational numbers. Then M is an ordered semiring. Define a map $d : M \to M$ by $d\begin{pmatrix} a & b \\ b & a \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2b & 0 \end{pmatrix}$ is a right derivation but not a derivation of M.*

THEOREM 3.10. *Let d be a right derivation of additively cancellative commutative idempotent non-negatively ordered semiring M. Define a set $Fix_d(M) = \{x \in M | d(x) = x\}$. Then $Fix_d(M)$ is an ideal of M.*

PROOF. Let d be a right derivation of additively cancellative commutative idempotent non-negatively ordered semiring M. Suppose $x, y \in Fix_d(M)$. Then

$$\begin{aligned} d(x) &= x, d(y) = y \\ d(x+y) &= d(x) + d(y) = x + y. \end{aligned}$$

Therefore $x + y \in Fix_d(M)$

$$\begin{aligned} d(xy) &= d(x)y + d(y)x \\ &= xy + yx \\ &= xy + xy = xy. \end{aligned}$$

Therefore $Fix_d(M)$ is a semiring of M. Suppose $x \leqslant y$ and $y \in Fix_d(M)$.

$$\begin{aligned} x &\leqslant y \\ \Rightarrow x + y &\leqslant y + y \\ \Rightarrow x + y &\leqslant y \leqslant x + y. \end{aligned}$$

Therefore $x + y = y$

$$\begin{aligned} \Rightarrow d(x+y) &= x+y \\ &\Rightarrow d(x)+d(y) = x+y \\ &\Rightarrow d(x)+y = x+y. \end{aligned}$$

Therefore $d(x) = x$. Therefore $d(x) = x$. Hence $Fix_d(M)$ is a k ideal of M. □

THEOREM 3.11. *Let d be a right derivation of ordered semiring M and I, J be any two d ideals of M and $I \subseteq J$. Then $d(I) \subseteq d(J)$*

PROOF. Suppose I, J be any two d ideals of ordered semiring M and $I \subseteq J$ and $x \in d(I)$. Then there exists $y \in I$ such that $x = d(y)$. Since $y \in I$ and $I \subseteq J, y \in J$. Therefore $x \in d(J)$. Hence $d(I) \subseteq d(J)$ □

THEOREM 3.12. *Let M be an ordered semiring and I, J be d ideals of M. Then $I + J$ is also a d ideal of M.*

PROOF. Let M be an ordered semiring and I, J be d ideals of M. Then I and J are subsets of $I + J$. Therefore

$$\begin{aligned} I &= d(I) \subseteq d(I+J) \\ J &= d(J) \subseteq d(I+J) \\ I+J &\subseteq d(I+J) \subseteq d(I)+d(J) = I+J \\ \Rightarrow d(I+J) &= I+J. \end{aligned}$$

Hence $I + J$ is also a d ideal of M. □

THEOREM 3.13. *Let M be a negatively ordered semiring with unity 1 and d be a right derivation of M. Then*
(i)$d(1)x \leqslant d(x)$
(ii) If $d(1) = 1$ then $x \leqslant d(x)$

PROOF. Let M be a negatively ordered semiring with unity $1, d$ be a right derivation of M and $x \in M$. Then there exists such that $x_1 = x$ and $1x = x$.

$$\begin{aligned} d(x) &= d(x1) \\ &= d(x)1 + d(1)x \\ \Rightarrow d(x)x &\leqslant d(x)1 + d(1)x = d(x). \end{aligned}$$

Suppose $d(1) = 1$ then $d(1)x \leqslant d(x)$. Therefore $1x \leqslant xd(x) \Rightarrow x \leqslant d(x)$. Hence the theorem. □

THEOREM 3.14. *Let M be negatively idempotent ordered semiring with unity and d be a right derivation. Then $d(1) = 1$ if and only if $d(x) = x$.*

PROOF. Let M be negatively idempotent ordered semiring with unity and d be a right derivation. Suppose $x \in M$.

$$\begin{aligned} d(x) &= d(xax) \\ &= d(x)x + d(x)x \\ &= d(x)x \leqslant x \\ d(x) &\leqslant x. \end{aligned}$$

Suppose $d(1) = 1$. By Theorem [3.13], we have $x \leqslant d(x)$. Therefore $d(x) = x$. Converse is obvious. □

THEOREM 3.15. *Let M be an ordered semiring and $d : M \to M$ be a right derivation. Then for an element $a \in M, d((a)^n)a = nd(a)(a)^{n-1}a$*

PROOF. Let M be an ordered semiring and $d : M \to M$ be a right derivation. We prove this result by mathematical induction. Let d be a right derivation on $M, a \in M$ and Suppose $n = 1$.

$$\begin{aligned} d(aa) &= d(a)a + d(a)a \\ &= 2d(a)a. \end{aligned}$$

Suppose that the statement is true for n. i.e. $d((a)^n)a) = nd(a)(a)^{n-1}a$. Then

$$\begin{aligned} d((a)^{n+1}a) &= nd(a)(a)^{n-1}a \\ &= d((a)^n)a)a + d(a)(a)^n)a \\ &= nd(a)(a)^{n+1}a + d(a)(a)^n a \\ &= (n+1)d(a)(a)^n a. \end{aligned}$$

Hence by mathematical induction, the theorem is true for all $n \in N$. □

DEFINITION 3.3. *An ordered semiring M is called a prime ordered semiring if $aMb = 0$ then $a = 0$ or $b = 0$. We write $[xy]$ for $xy - yx$.*

THEOREM 3.16. *Let M be a additively commutative and cancellative prime ordered semring and d be a non-zero right derivation of M. Then M is commutative ordered semiring.*

PROOF. Let M be a additively commutative and cancellative prime ordered semring and $d : M \to M$ be a non-zero right derivation of M. Suppose $a, b \in M$.

$$\begin{aligned} d(aba) &= d(a)ba + d(ba)a \\ &= d(a)ba + (d(b)a + d(a)b)a. \\ &= d(a)ba + d(b)aa + d(a)ba \ \cdots (1). \\ d(aba) &= d(ab)a + d(a)(ab) \\ &= d(a)b + d((b)a)a + d(a)ab \\ &= d(a)ba + d(b)aa + d(a)ab \ \cdots (2). \end{aligned}$$

From (1) and (2),

$$d(a)ba = d(a)ab$$

$$\Rightarrow d(a)[ab] = 0, \text{for all} a, b \in M.$$

Replacing b by cb, we get $d(a)c[ab] = 0$, for all $a, b \in M$, $\Rightarrow d(a)M[ab] = 0$, for all $a, b \in M$. Since $d(a) \neq 0$, we get $[ab] = 0$, for all $a, b \in M$. Hence M is a commutative ordered semiring. □

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