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GENERALIZED MICRO Δ-LOCALLY CLOSED SETS IN MICRO TOPOLOGICAL SPACES

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Abstract. The purpose of this paper is to introduce a new class of sets called generalized micro locally closed sets in micro topological spaces. We also introduce generalized micro locally continuous map, generalized micro locally closed irresolute map, and study some of their properties.

1. Introduction

Several notions of open-like and closed-like sets in nano topological spaces were introduced and studied. The beginning was with M. Lellis Thivagar and Carmel Richard who initiated the notion of nano forms of weakly open sets and nano continuity, [**6**, **7**]. Several notions of open-like and closed-like sets in micro topological spaces were introduced and studied. The beginning was with S. Chandrasekar who initiated the notion of micro forms of open sets, [**1**, **2**]. We introduced and studied the notion of micro Δ-open sets in micro topological spaces, [**4**]. The concept of micro continuity in micro topological spaces was extended to generalized micro Δ-continuity, [**5**].

A set in a topological space is called Δ-open if it is the symmetric difference of two open sets. The notion of Δ-open sets appeared in [**8**] and in [**3**]. However, it was pointed out in [**8**] and in [**3**] that the notion of Δ-open sets is due to a preprint by M. Veera Kumar. The complement of a Δ-open set is Δ-closed.

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A set in a micro topological space is called $m\Delta$-open if it is the symmetric difference of two micro open sets were initiated, [**4**].

Preliminary concepts required in our work are briefly recalled in section 2. In section 3, we introduced the classes of $m\Delta$-lc-set, $gm\Delta$-lc-set, $gm\Delta$-lc*-sets, $gm\Delta$-lc**-sets and study some of its basic properties. In section 4, Finally we introduced and studied $m\Delta$LC-continuous, $Gm\Delta$LC-continuous map and $Gm\Delta$LC-irresolute map.

2. Preliminaries

Definition 2.1. *[4] A subset S of a space (S, $\tau_R(X)$) is said to be micro Δ-open set (in short, mΔ-open) if S = (A − B) ∪ (B − A), where A and B are micro-open subsets in X. The complement of micro-Δ-open sets is called micro -Δ-closed sets.*

Definition 2.2. *[4] The micro interior of a set A is denoted by micro Δ-int(A) (briefly, mΔ-int(A)) and is defined as the union of all mΔ open sets contained in A. i.e., mΔ-int(A) = ∪ {G : G is mΔ-open and G ⊆ A }.*

Definition 2.3. *[4] The micro closure of a set A is denoted by micro Δ-cl(A) (briefly, mΔ-cl(A)) and is defined as the intersection of all mΔ-closed sets containing A. i.e., mΔ-cl(A) = ∩ {F : F is mΔ-closed and A ⊆ F}.*

Definition 2.4. *[5] A subset U of a space (U, $\tau_R(X)$, $\mu_R(X)$) is called a generalized micro Δ-closed (briefly, gmΔ-closed) set if mΔcl(A) ⊆ T whenever A ⊆ T and T is mΔ-open in (U, $\tau_R(X)$, $\mu_R(X)$).*
The complement of gmΔ-closed set is called gmΔ-open set.

Proposition 2.1. *[5] Every mΔ-closed set is gmΔ-closed.*

Proposition 2.2. *[5] Every mΔ-open set is gmΔ-open set but not conversely.*

3. Generalized $m\Delta$-locally closed sets

Definition 3.1. *A subset A of an micro topological space (U, $\tau_R(X)$, $\mu_R(X)$) is called an mΔ-locally closed (briefly, mΔ-lc) sets if A = S ∩ G where S is mΔ-open and G is mΔ-closed.*
The class of all mΔ-locally closed sets in a micro topological space (U, $\tau_R(X)$, $\mu_R(X)$) is denoted by mΔLC(X).

Definition 3.2. *A subset A of an micro topological space (U, $\tau_R(X)$, $\mu_R(X)$) is called an generalized mΔ-locally closed (briefly, gmΔ-lc) sets if A = E ∩ F where E is gmΔ-open and F is gmΔ-closed.*
The class of all gmΔ-locally closed sets in micro topological spaces (U, $\tau_R(X)$, $\mu_R(X)$) is denoted by GmΔLC(X).

Proposition 3.1. *Every mΔ-closed (resp. mΔ-open) set is mΔ-lc-set but not conversely.*

PROOF. It follows from Definition 3.1. □

EXAMPLE 3.1. *Let* $U = \{1, 2, 3\}$ *with* $U/R = \{\{1\}, \{2, 3\}\}$ *and* $X = \{1\}$. *The nano topology* $\tau_R(X) = \{\phi, \{1\}, U\}$. *If* $\mu = \{1, 3\}$ *then the micro topology* $\mu_R(X) = \{\phi, \{1\}, \{1, 3\}, U\}$. *Then* $m\Delta$*-open sets are* ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 3\}$, $\{2, 3\}$, U; $m\Delta$*-closed sets are* ϕ, $\{1\}$, $\{2\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, U; $m\Delta lc$*-sets are* ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, U. *Here, the set* $\{1, 2\}$ *is* $m\Delta$*-lc set but it is not* $m\Delta$*-open and the set* $\{3\}$ *is* $m\Delta$*-lc set but it is not* $m\Delta$*-closed in* $(U, \tau_R(X), \mu_R(X))$.

PROPOSITION 3.2. *Every* $gm\Delta$*-closed (resp.* $gm\Delta$*-open) set is* $gm\Delta$*-lc-set but not conversely.*

PROOF. It follows from Definition 3.2. □

EXAMPLE 3.2. *Let* $(U, \tau_R(X), \mu_R(X))$ *as int the Example 3.1. Then* $gm\Delta$*-open sets are* ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 3\}$, $\{2, 3\}$, U; $gm\Delta$*-closed sets are* ϕ, $\{1\}$, $\{2\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, U; $gm\Delta$*-lc-sets are* ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, U. *Here, the set* $\{1,2\}$ *is* $gm\Delta$*-lc set but it is not* $gm\Delta$*-open and the set* $\{3\}$ *is* $gm\Delta$*-lc set but it is not* $gm\Delta$*-open in* $(U, \tau_R(X), \mu_R(X))$.

PROPOSITION 3.3. *Every* $m\Delta$*-lc-set is* $gm\Delta$*-lc-set but not conversely.*

PROOF. It follows from Proposition 2.1 and 2.2. □

EXAMPLE 3.3. *Let* $U = \{1, 2, 3\}$ *with* $U/R = \{\{1, 2, 3\}\}$ *and* $X = \{2, 3\}$. *The nano topology* $\tau_R(X) = \{\phi, U\}$. *If* $\mu = \{1, 3\}$ *then the micro topology* $\mu_R(X) = \{\phi, \{1, 3\}, U\}$. *Then* $m\Delta$*-lc-sets are* ϕ, $\{2\}$, $\{1, 3\}$, U, $gm\Delta$*-lc-sets are* ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, U. *Here, the set* $\{2, 3\}$ *is* $gm\Delta$*-lc set but it is not* $m\Delta$*-lc set in* $(U, \tau_R(X), \mu_R(X))$.

DEFINITION 3.3. *A space* $(U, \tau_R(X), \mu_R(X))$ *is called a* $T_\Delta 1/2$*-space if every* $mg\Delta$*-closed set is* $m\Delta$*-closed.*

THEOREM 3.1. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a* $T_\Delta 1/2$*-space. Then* $Gm\Delta LC(X) = m\Delta LC(X)$.

PROOF. Since every $gm\Delta$-open set is $m\Delta$-open and every $gm\Delta$-closed set is $m\Delta$-closed in (U, $\tau_R(X)$, $\mu_R(X)$), G$m\Delta$LC(X) $\subseteq$ $m\Delta$LC(X) and hence G$m\Delta$LC(X) = $m\Delta$LC(X). □

DEFINITION 3.4. *A subset A of a space* $(U, \tau_R(X), \mu_R(X))$ *is called*

(1) $gm\Delta$*-lc***-set if* $A = O \cap P$, *where O is* $gm\Delta$*-open in* $(U, \tau_R(X), \mu_R(X))$ *and P is* $m\Delta$*-closed in* $(U, \tau_R(X), \mu_R(X))$.
(2) $gm\Delta$*-lc****-set if* $A = R \cap S$, *where R is* $m\Delta$*-open in* $(U, \tau_R(X), \mu_R(X))$ *and S is* $gm\Delta$*-closed in* $(U, \tau_R(X), \mu_R(X))$.

The class of all $gm\Delta$*-lc** *(resp.* $gm\Delta$*-lc****) sets in a micro topological space* $(U, \tau_R(X), \mu_R(X))$ *is denoted by* $gm\Delta LC^*(X)$ *(resp.* $gm\Delta LC^{**}(X)$*).*

PROPOSITION 3.4. *Every* $m\Delta$*-lc-set is* $gm\Delta$*-lc***-set but not conversely.*

Proof. It follows from Definitions 3.1 and 3.4 (1). □

Proposition 3.5. *Every $m\Delta$-lc-set is $gm\Delta$-lc**-set but not conversely.*

Proof. It follows from Definitions 3.1 and 3.4(2). □

Proposition 3.6. *Every $gm\Delta$-lc*-set is $gm\Delta$-lc-set but not conversely.*

Proof. It follows from Definitions 3.2 and 3.4(1). □

Proposition 3.7. *Every $gm\Delta$-lc**-set is $gm\Delta$-lc-set but not conversely.*

Proof. It follows from Definitions 3.2 and 3.4 (2). □

Theorem 3.2. *For a subset A of (U, $\tau_R(X)$, $\mu_R(X)$) the following statements are equivalent:*

(1) $A \in Gm\Delta LC(X)$,
(2) $A = S \cap gm\Delta\text{-}cl(A)$ for some $gm\Delta$-open set S,
(3) $gm\Delta\text{-}cl(A) - A$ is $gm\Delta$-closed,
(4) $A \cup (gm\Delta\text{-}cl(A))^c$ is $gm\Delta$-open,
(5) $A \subseteq gm\Delta\text{-}int(A \cup (gm\Delta\text{-}cl(A))^c)$.

Proof. (1) ⇒ (2). Let A ∈ G$m\Delta$LC(X). Then A = S ∩ G where S is $gm\Delta$-open and G is $gm\Delta$-closed. Since A ⊆ G, $gm\Delta$-cl(A) ⊆ G and so S ∩ $gm\Delta$-cl(A) ⊆ A. Also A ⊆ S and A ⊆ $gm\Delta$-cl(A) implies A ⊆ S ∩ $gm\Delta$-cl(A) and therefore A = S ∩ $gm\Delta$-cl(A).

(2) ⇒ (3). A = S ∩ $gm\Delta$-cl(A) implies $gm\Delta$-cl(A) − A = $gm\Delta$-cl(A) ∩ S^c which is $gm\Delta$-closed since S^c is $gm\Delta$-closed and $gm\Delta$-cl(A) is $gm\Delta$-closed.

(3) ⇒ (4). A ∪ ($gm\Delta$-cl(A))^c = ($gm\Delta$-cl(A) − A)^c and by assumption, ($gm\Delta$-cl(A) − A)^c is $gm\Delta$-open and so is A ∪ ($gm\Delta$-cl(A))^c.

(4) ⇒ (5). By assumption, A ∪ ($gm\Delta$-cl(A))^c = $gm\Delta$-int(A ∪ ($gm\Delta$-cl(A))^c) and hence A ⊆ $gm\Delta$-int(A ∪ ($gm\Delta$-cl(A))^c).

(5) ⇒ (1). By assumption and since A ⊆ $gm\Delta$-cl(A), A = $gm\Delta$-int(A ∪ ($gm\Delta$-cl(A))^c) ∩ $gm\Delta$-cl(A). Therefore, A ∈ $gm\Delta$LC(X). □

Theorem 3.3. *For a subset A of (U, $\tau_R(X)$, $\mu_R(X)$), the following statements are equivalent:*

(1) $A \in gm\Delta LC^{*}(U)$,
(2) $A = S \cap m\Delta cl(A)$ for some $gm\Delta$-open set S,
(3) $m\Delta cl(A) - A$ is $gm\Delta$-closed,
(4) $A \cup (m\Delta cl(A))^c$ is $gm\Delta$-open.

Proof. (1) ⇒ (2). Let A ∈ $gm\Delta$LC *(U). There exist an $gm\Delta$-open set S and a $m\Delta$-closed set G such that A = S ∩ G. Since A ⊆ S and A ⊆ $m\Delta$cl(A), A ⊆ S ∩ $m\Delta$cl(A). Also since $m\Delta$cl(A) ⊆ G, S ∩ $m\Delta$cl(A) ⊆ S ∩ G = A. Therefore A = S ∩ $m\Delta$cl(A).

(2) ⇒ (1). Since S is $gm\Delta$-open and $m\Delta$cl(A) is a $m\Delta$-closed set, A = S ∩ $m\Delta$cl(A) ∈ $gm\Delta$LC *(U).

(2) ⇒ (3). Since $m\Delta$cl(A) − A = $m\Delta$cl(A) ∩ S^c, $m\Delta$cl(A) − A is $gm\Delta$-closed since S^c is $gm\Delta$-closed.

(3) ⇒ (2). Let S = ($m\Delta$cl(A) − A)^c. Then by assumption S is $gm\Delta$-open in (U, $\tau_R(X)$, $\mu_R(X)$) and A = S ∩ $m\Delta$cl(A).

(3) ⇒ (4). Let G = $m\Delta$cl(A) − A. Then G^c = A ∪ ($m\Delta$cl(A))^c and A ∪ ($m\Delta$cl(A))^c is $gm\Delta$-open.

(4) ⇒ (3). Let S = A ∪ ($m\Delta$cl(A))^c. Then S^c is $gm\Delta$-closed and S^c = $m\Delta$cl(A) − A and so $m\Delta$cl(A) − A is $gm\Delta$-closed. □

THEOREM 3.4. *Let A be a subset of (U, $\tau_R(X)$, $\mu_R(X)$).*
Then A ∈ $gm\Delta LC^{**}$(X) if and only if A = S ∩ $gm\Delta$-cl(A) for some $m\Delta$-open set S.

PROOF. (1) ⇒ (2). Let A ∈ $gm\Delta$LC**(X). Then A = S ∩ G where S is $m\Delta$-open and G is $gm\Delta$-closed. Since A ⊆ G, $gm\Delta$-cl(A) ⊆ G. We obtain A = A ∩ $gm\Delta$-cl(A) = S ∩ G ∩ $gm\Delta$-cl(A) =S ∩ $gm\Delta$-cl(A).

(2) ⇒ (1). Since S is $m\Delta$-open and $gm\Delta$-cl(A) is a $gm\Delta$-closed set, A = S ∩ $gm\Delta$-cl(A) ∈ $gm\Delta$LC**(X). □

COROLLARY 3.1. *Let A be a subset of (U, $\tau_R(X)$, $\mu_R(X)$).*
If A ∈ $gm\Delta LC^{**}$(X), then $gm\Delta$cl(A) − A is $gm\Delta$-closed and A ∪ ($gm\Delta$-cl(A))^c is $gm\Delta$-open.

PROOF. Let A ∈ $gm\Delta$LC**(X). Then by Theorem 3.4, A = S ∩ $gm\Delta$-cl(A) for some $m\Delta$-open set S and $gm\Delta$-cl(A) − A = $gm\Delta$-cl(A) ∩ S^c is $gm\Delta$-closed in (U, $\tau_R(X)$, $\mu_R(X)$). If G = $gm\Delta$-cl(A) − A, then G^c = A ∪ ($gm\Delta$-cl(A))^c and G^c is $gm\Delta$-open and so is A ∪ ($gm\Delta$-cl(A))^c. □

4. $G\Delta$LC -continuous and $G\Delta$LC -irresolute maps

DEFINITION 4.1. *A map f : (U, $\tau_R(X)$, $\mu_R(X)$) → (V, $\tau_R(X)'$, $\mu_R(X)'$) is said to be mΔ locally closed-continuous (briefly, mΔLC-continuous) if f^{-1}(V) is mΔLC-set in (U, $\tau_R(X)$, $\mu_R(X)$) for every mΔ-open set V of (V, $\tau_R(X)'$, $\mu_R(X)'$).*

DEFINITION 4.2. *A map f : (U, $\tau_R(X)$, $\mu_R(X)$) → (V, $\tau_R(X)'$, $\mu_R(X)'$) is said to be GΔLC-continuous (resp. GΔLC^*-continuous, GΔLC^{**}-continuous) if f^{-1}(V) is GΔLC-set (resp. GΔLC^*-set, GΔLC^{**}-set) in (U, $\tau_R(X)$, $\mu_R(X)$) for every mΔ-open set V of (V, $\tau_R(X)'$, $\mu_R(X)'$).*

THEOREM 4.1. *Let f : (U, $\tau_R(X)$, $\mu_R(X)$) → (V, $\tau_R(X)'$, $\mu_R(X)'$) be a map. Then*

(1) *If f is mΔ-continuous, then it is mΔLC-continuous.*
(2) *If f is mΔ-continuous, then it is GΔLC-continuous.*
(3) *If f is gΔ-continuous, then it is GΔLC-continuous.*

PROOF. (1) It is an immediate consequence of Proposition 3.1.
(2) It is an immediate consequence of Proposition 3.1 and 3.3.
(3) It is an immediate consequence of Propositions 3.2. □

Theorem 4.2. *Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) be a map. Then*

1. *If f is mΔLC-continuous, then it is GΔLC-continuous.*
2. *If f is mΔLC-continuous, then it is GΔLC^{*}-continuous.*
3. *If f is mΔLC-continuous, then it is GΔLC^{**}-continuous.*
4. *If f is GΔLC^{*}-continuous, then it is GΔLC-continuous.*
5. *If f is GΔLC^{**}-continuous, then it is GΔLC-continuous.*

Proof. (1) It is an immediate consequence of Proposition 3.3.
(2) It is an immediate consequence of Proposition 3.4.
(3) It is an immediate consequence of Propositions 3.5.
(4) It is an immediate consequence of Propositions 3.6.
(5) It is an immediate consequence of Propositions 3.2. □

Definition 4.3. *A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) is said to be GΔLC -irresolute (resp. GΔLC^{*}-irresolute, GΔLC^{**}-irresolute) if $f^{-1}(V)$ is GΔLC -set (resp. GΔLC^{*}-set, GΔLC^{**}-set) in (U, $\tau_R(X)$, $\mu_R(X)$) for every GΔLC -set (resp. GΔLC^{*}-set, GΔLC^{**}-set) V of (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$).*

Theorem 4.3. *Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) be a map. Then*

1. *If f is GΔLC -irresolute then it is GΔLC -continuous.*
2. *If f is GΔLC^{*}-irresolute then it is GΔLC^{*}-continuous.*
3. *If f is GΔLC^{**}-irresolute then it is GΔLC^{**}-continuous.*

Proof. (1) Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) be a $G\Delta$LC -irresolute map. Let V be a $m\Delta$-open set of (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$). Since every $m\Delta$-open set is $g\Delta$-open and $g\Delta$-open set is $g\Delta$-lc-set [by the Proposition 2.2 and Proposition 3.2], V is $G\Delta$LC -set of (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$). Since f is $G\Delta$LC -irresolute, then f^{-1}(V) is a $G\Delta$LC -set of (U, $\tau_R(X)$, $\mu_R(X)$). Therefore f is $G\Delta$LC -continuous.
(2) Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) be a $G\Delta$LC *-irresolute map. Let V be a $m\Delta$-open set of (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$). Since every $m\Delta$-open set is $m\Delta$-lc set and $m\Delta$-lc-set is $g\Delta$-lc*-set [by Proposition 3.1 and Proposition 3.4], V is $G\Delta$LC *-set of (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$). Since f is $G\Delta$LC *-irresolute, then f^{-1}(V) is a $G\Delta$LC *-set of (U, $\tau_R(X)$, $\mu_R(X)$). Therefore f is $G\Delta\, LC^{*}$-continuous.
(3) Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) be a $G\Delta$LC **-irresolute map. Let V be a $m\Delta$-open set of (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$). Since every $m\Delta$-open set is $m\Delta$-lc-set and $m\Delta$-lc-set is $g\Delta$-lc**-set [by Proposition 3.1 and Proposition 3.5], A is $G\Delta$LC **-set of (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$). Since f is $G\Delta$LC **-irresolute, then f^{-1}(V) is a $G\Delta\, LC^{**}$-set of (U, $\tau_R(X)$, $\mu_R(X)$). Therefore f is $G\Delta\, LC^{**}$-continuous. □

Theorem 4.4. *Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) and g : (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) $\rightarrow$ (Z, γ) be any two maps. Then*

1. *$g \circ f$ is GΔLC-continuous if g is mΔ-continuous and f is GΔLC-continuous.*

(2) *$g \circ f$ is $G\Delta LC$-irresolute if both f and g are $G\Delta LC$-irresolute.*
(3) *$g \circ f$ is $G\Delta LC$-continuous if g is $G\Delta LC$-continuous and f is $G\Delta LC$-irresolute.*

Proof. (1) Since g is a $m\Delta$-continuous from (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) $\rightarrow$ (Z, γ), for any $m\Delta$-open set z as a subset of Z, we get $g^{-1}(z)$ = G is a $m\Delta$-open set in (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) . As f is a $G\Delta$LC-continuous map. We get $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G)$ = S and S is a $G\Delta$LC-set in (U, $\tau_R(X)$, $\mu_R(X)$), since every $m\Delta$-open set is $g\Delta$-open set and $g\Delta$-open set is $g\Delta$-lc-set [by the Proposition 2.2 and Proposition 3.2]. Hence $(g \circ f)$ is a $G\Delta$LC-continuous map.
(2) Consider two $G\Delta$LC-irresolute maps, f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) and g : (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) $\rightarrow$ (Z, γ) is a $G\Delta$LC-irresolute maps. As g is consider to be a $G\Delta$LC-irresolute map, by Definition 4.3, for every $g\Delta$-lc-set z $\subseteq$ (Z, γ), $g^{-1}(z)$ = G is a $g\Delta$-lc-set in (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$). Again since f is $G\Delta$LC-irresolute, $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G)$ = S and S is a $g\Delta$-lc-set in (U, $\tau_R(X)$, $\mu_R(X)$). Hence $(g \circ f)$ is a $G\Delta$LC-irresolute map.
(3) Let g be a $G\Delta$LC-continuous map from (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) $\rightarrow$ (Z, γ) and z subset of Z be a $m\Delta$-open set. Therefore $g^{-1}(z)$ = G is a $g\Delta$-lc-set in (V, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$), since every $m\Delta$-open set is $g\Delta$-open set and $g\Delta$-open set is $g\Delta$-lc-set [by the Proposition 2.2 and Proposition 3.2]. Also since f is $G\Delta$LC-irresolute, we get $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G)$ = S and S is a $g\Delta$-lc-set in (U, $\tau_R(X)$, $\mu_R(X)$). Hence $(g \circ f)$ is a $G\Delta$LC-continuous map. □

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