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TRI-IDEALS OF SEMIGROUPS

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Abstract. In this paper, we introduce the notion of a tri-ideal as a generalization of ideals, right ideals, left ideals, bi ideals, quasi ideals, interior ideals, bi quasi ideals, bi interior ideals, and quasi interior ideals of a semigroup. We study the properties of tri-ideals of a semigroup and characterize the tri-simple semigroup using tri-ideals of a semigroup.

1. Introduction

Semigroup, as the basic algebraic structure was used in the areas of theoretical computer science as well as in the solutions of graph theory, optimization theory and in particular for studying automata, coding theory and formal languages. The notion of ideals was introduced by Dedekind for the theory of algebraic numbers.The notion of ideals was generalized by E. Noether for associative rings. The one and two sided ideals introduced by her, are still central concepts in ring theory. We know that the notion of a one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi ideals are generalization of left ideals and right ideals whereas the bi-ideals are generalization of quasi ideals.

Ideals play an important role in advance studies and uses of algebraic structures. Generalization of ideals in algebraic structures is necessary for further study of algebraic structures. Many mathematicians proved important results and charecterizations of algebraic structures by using the concept and the properties of generalization of ideals in algebraic structures.

In 1952, the concept of bi-ideals was introduced by Good and Hughes [1]for semigroups. The notion of bi-ideals in rings and semirings were introduced by

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Lajos and Szasz [9,10].Bi-ideal is a special case of (m-n) ideal. Steinfeld [17] first introduced the notion of quasi ideals for semirings and then for rings. Iseki [5,6,7,8] introduced the concept of quasi ideal for a semiring. Quasi ideals, bi-ideals in $\Gamma-$semirings studied by Jagtap and Pawar [7,8]. The author [**11**–**20**]introduced the notions of bi quasi ideals, bi interior ideals, quasi interior ideals, bi quasi interior ideals, tri quasi ideals, and weak interior ideals of semigroups, semirings, Gamma semigroups and Gamma -semirings. In this paper, as a further generalization of ideals, we introduce the notion of a tri-ideal and study the properties of tri-ideals of a semigroup and a simple semigroup.

2. Preliminaries

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. *A semigroup is an algebraic system $(S,.)$ consisting of a non-empty set S together with an associative binary operation $'' \cdot ''$.*

Definition 2.2. *A non-empty subset A of a semigroup M is called*

(i) *a subsemigroup of M $AA \subseteq A$.*
(ii) *a quasi ideal of M if A is $AM \cap MA \subseteq A$.*
(iii) *a bi-ideal of M if A is a subsemigroup of M and $AMA \subseteq A$.*
(iv) *an interior ideal of M if A is a subsemigroup of M and $MAM \subseteq A$.*
(v) *a left (right) ideal of M if $MA \subseteq A(AM \subseteq A)$.*
(vi) *an ideal if $AM \subseteq A$ and $MA \subseteq A$.*
(vii) *a bi-quasi-interior ideal of M if A is a subsemigroup of M and $AMAMAB \subseteq A$.*
((viii) *a bi-interior ideal of M if A is a subsemigroup of M and $MAM \cap AMA \subseteq A$.*
((ix) *a left bi-quasi ideal (right bi-quasi ideal) of M if A is a subsemigroup and $MA \cap AMA \subseteq A$ $(AM \cap AMA \subseteq A)$.*
((x) *a left quasi-interior ideal (right quasi-interior ideal) of M if A is a subsemi group of $(M,+)$ and $MAMA \subseteq A$ $(AMAM \subseteq A)$.*

Definition 2.3. *Let S be a semigroup. An element $1 \in S$ is said to be unity if $x1 = 1x = x$ for all $x \in S$.*

Definition 2.4. *A semigroup M is a left (right) simple semigroup if M has no proper left (right) ideal of M*

Definition 2.5. *A semigroup M is a bi-quasi simple semigroup if M has no proper bi-quasi ideal of M*

Definition 2.6. *A semigroup M is said to be simple semigroup if M has no proper ideals.*

3. Tri-ideals of semigroups

In this section, we introduce the notion of a tri-ideal as a generalization of ideals, right ideals, left ideals, bi ideals, quasi ideals, interior ideals of a semigroup and study the properties of semigroups. Through out this paper M is a semigroup with unity element.

Definition 3.1. *A non-empty subset B of a semigroup M is said to be right tri-ideal of M if B is a subsemigroup of M and $BBMB \subseteq B$.*

Definition 3.2. *A non-empty subset B of a semigroup M is said to be left tri-ideal of M if B is a subsemigroup of M and $BMBB \subseteq B$.*

Definition 3.3. *A non-empty subset B of a semigroup M is said to be tri-ideal of M if B is a subsemigroup of M and $B$is a left and a right tri-ideal of M.*

Remark: A tri-ideal of a semigroup M need not be quasi-ideal, interior ideal, bi-interior ideal. and bi-quasi ideal of a semigroup M.

example 3.1. *Let $M = \{a, b, c, d, e\}$, define the binary operation "$\cdot$" with the following table*

$\cdot$	a	b	c	d	e
a	a	a	a	a	a
b	a	b	a	d	d
c	a	d	c	d	e
a	a	d	a	d	d
a	a	d	e	d	e

Therefore M is a semigroup.
Then $B = \{a, c\}$ is a left tri-ideal. B is not a left ideal, not an interior ideal, and not a quasi-interior ideal.

In the following theorem, we mention some important properties and we omit the proofs since they are straight forward.

Theorem 3.1. *Let M be a semigroup. Then the following are hold.*

(1) *Every left ideal is a tri-ideal of M.*
(2) *Every right ideal is a tri-ideal of M.*
(3) *Every quasi ideal is a tri-ideal of M.*
(4) *Every ideal is a tri-ideal of M.*
(5) *Intersection of a right ideal and a left ideal of M is a tri-ideal of M.*
(6) *If L is a left ideal and R is a right ideal of a semigroup M then $B = RL$ is a tri-ideal of M.*
(7) *If B is a tri-ideal and T is a subsemigroup of M then $B \cap T$ is a tri-ideal of ring M.*
(8) *Let M be a semigroup and B be a subsemigroup of M. If $MMM \subseteq B$ then B is a left tri-ideal of M.*
(9) *Let M be a semigroup and B be a subsemigroup of M.If $MMMB \subseteq B$ and$BMMM \subseteq B$ then B is a tri-ideal of M.*
(10) *Intersection of a right tri-ideal and a left tri-ideal of M is a tri-ideal of M.*
(11) *If L is a left ideal and R is a right ideal of M then $B = R \cap L$ is a tri-ideal of M.*

Theorem 3.2. *If B be a left bi-quasi ideal of a semigroup M, then B is a tri-ideal of M.*

Proof. Suppose B is a left bi-quasi of the semigroup M. Then $BMBB \subseteq MB$. We have $BMBB \subseteq BMB$ Therefore $BMBB \subseteq MB \cap BMB \subseteq B$ Hence B is a left tri-ideal of M. Similarly we can show that B is a right tri-ideal of M. Hence B is a tri-ideal of M. □

Corollary 3.1. *If B be a right bi-quasi ideal of a semigroup M, then B is a tri- ideal of M.*

Corollary 3.2. *If B be a bi-quasi ideal of a semigroup M, then B is a tri-ideal of M.*

Theorem 3.3. *If B be a bi–interior ideal of a semigroup M, then B is a left tri- ideal of M.*

Proof. Suppose B is a bi–interior ideal of the semigroup M.Then
$MBM \cap BMB \subseteq B$, $BMBB \subseteq MBM \cap BMB \subseteq B$ Hence B is a left tri ideal of M. □

Corollary 3.3. *If B be a bi–interior ideal of a semigroup M, then B is a right tri-ideal of M.*

Corollary 3.4. *If B be a bi–interior ideal of a semigroup M, then B is a tri-ideal of M.*

Theorem 3.4. *If B is a subsemigroup of a semigroup M and $MBB \subseteq B$, then B is a left tri- ideal of M.*

Theorem 3.5. *Every bi-ideal of a semigroup M is a left tri-ideal of a semigroup M.*

Proof. Let B be a bi-ideal l of the semigroup M.
Then $BMBB \subseteq BMB \subseteq B$. Therefore $BMBB \subseteq B$.
Hence every bi-ideal of a semigroup M is a left tri-ideal of a semigroup M. □

Corollary 3.5. *Every bi-ideal right tri-ideal of a semigroup M is a tri-ideal of a semigroup M.*

Corollary 3.6. *Every bi-ideal of a semigroup M is a tri-ideal of a semigroup M.*

Theorem 3.6. *Every left tri-ideal of a semigroup M is a bi-quasi interior ideal of a semigroup M.*

Proof. Let B be a left tri-ideal of the semigroup M.
Then $BMBB \subseteq B$. Therefore $BMBMB \subseteq B$. This completes the proof. □

Corollary 3.7. *Every right tri-ideal of a semigroup M is a bi-quasi interior ideal of semigroup M.*

Corollary 3.8. *Every tri-ideal of a semigroup M is a bi-quasi interior ideal of semigroup M.*

Theorem 3.7. *Every interior ideal of a semigroup M is a left tri-ideal of M.*

Proof. Let I be an interior ideal of the semigroup M.
Then $IMII \subseteq MIM \subseteq I$.
Hence I is a left tri-ideal of semigroup M. □

Corollary 3.9. *Every interior ideal of a semigroup M is a right tri-ideal of M.*

Corollary 3.10. *Every interior ideal of a semigroup M is a tri-ideal of M.*

Theorem 3.8. *Let M be a semigroup and B be a subsemigroup of M.Then B is a left tri-ideal of M if and only if there exist left ideals L and R such that $RL \subseteq B \subseteq R \cap L$.*

Proof. Suppose B is a tri-ideal of the semigroup$M$.
Then $BMBB \subseteq B$. Let $R = BM$ and $L = MBB$.
Then R and L are a right ideal and a left ideal of M respectively.
Therefore $RL \subseteq B \subseteq R \cap L$.
Conversely suppose that there exist R and L are a right ideal and a left ideal of M respectively such that $R\Gamma L \subseteq B \subseteq R \cap L$ .Then $BMBB \subseteq (R \cap L)M(R \cap L)(R \cap L)$ $\subseteq RL \subseteq B$.
Hence B is a left tri-ideal of M. □

Corollary 3.11. *Let M be a semigroup and B be a subsemigroup of M.Then B is a right tri-ideal of M if and only if there exist right ideals L and R such that $RL \subseteq B \subseteq R \cap L$.*

Theorem 3.9. *The intersection of a left tri-ideal B of a semigroup M and a right ideal A of M is always a left tri-ideal of M.*

Proof. Suppose $C = B \cap A$.

$$\begin{aligned} & CMCC \subseteq BMBB \subseteq B \\ & CMCC \subseteq AMAA \subseteq A \text{ Since } A \text{ is a left ideal of } M \\ \text{Therefore } & CMCC \subseteq B \cap A = C. \end{aligned}$$

Hence the intersection of a left tri-ideal B of the semigroup M and a left ideal A of M is always a left tri-ideal of M. □

Corollary 3.12. *The intersection of a right tri-ideal B of a semigroup M and a right ideal A of M is always a right tri-ideal of M.*

Corollary 3.13. *The intersection of a tri-ideal B of a semigroup M and an ideal A of M is always a tri-ideal of M.*

Theorem 3.10. *Let A and C be left tri- ideals of a semigroup M, $B = AC$ and B is an additively subsemigroup of M. If $AA = A$ then B is a left tri-ideal of M.*

Proof. Let A and C be left tri- ideals of the semigroup M and $B = AC$. Then $BB = ACAC = ACAAC \subseteq AMAAC \subseteq AC = B$. Therefore $B = AC$ is a subsemigroup of M

$$\begin{aligned} BMBB &= ACMACAC \\ &\subseteq AMAC \subseteq AC = B. \end{aligned}$$

Hence B is a left tri-ideal of M. □

Theorem 3.11. *Let A and C be subsemigroups of a semigroup M and $B = AC$ and B is additively subsemigroup of M. If A is the left ideal of M, then B is a tri-ideal of M.*

Proof. Let A and C be subsemigroups of M and $B = AC$. Suppose A is the left ideal of M. Then $BB = ACAC \subseteq AC = B$.

$$\begin{aligned} BMBB &= ACMACAC \\ &\subseteq AC = B. \end{aligned}$$

Hence B is a left tri-ideal of M. □

Corollary 3.14. *Let A and C be subsemigroups of a semigroup M and $B = AC$and B is additively subsemigroup of M. If C is a right ideal then B is a right tri-ideal of M.*

Theorem 3.12. *Let M be a semigroup and T be a non-empty subset of M. If subsemigroup B of M containing $TMMT$ and $B \subseteq T$, then B is a left tri-ideal of semigroup M.*

Proof. Let B be a subsemigroup of M containing $MTMT$. Then

$$\begin{aligned} BMB &\subseteq TMTTT \\ &\subseteq B. \end{aligned}$$

Therefore $BMBB \subseteq B$.
Hence B is a left tri-ideal of M. □

Theorem 3.13. *Let B be a bi-ideal of a semigroup M and I be an interior ideal of M. Then $B \cap I$ is a left tri-ideal of M.*

Proof. Suppose B is a bi-ideal of M and I is an interior ideal of M. Obviously $B \cap I$ is subsemigroup of M. Then

$$(B \cap I)M(B \cap I)(B \cap I) \subseteq BMBB \subseteq B$$
$$(B \cap I)M(B \cap I)(B \cap I) \subseteq IMI \subseteq I$$

Therefore $(B \cap I)M(B \cap I)(B \cap I) \subseteq B \cap I$.
Hence $B \cap I$ is a left tri-ideal of M. □

Theorem 3.14. *Let B be a bi-ideal of a semigroup M and I be an interior ideal of M. Then $B \cap I$ is a right tri-ideal of M.*

Theorem 3.15. *Let M be a semigroup and T be a subsemigroup of M. Then every subsemigroup of T containing $TMTT$ is a left tri-ideal of M.*

Proof. Let C be a subsemigroup of T containing $TMTT$. Then

$$C\Gamma MCC \subseteq TMTT \subseteq C.$$

Hence C is a left tri-ideal of M . □

Theorem 3.16. *The intersection of* $\{B_\lambda \mid \lambda \in A\}$ *left tri-ideals of a semigroup* M *is a left tri-ideal of* M.

Proof. Let $B = \bigcap\limits_{\lambda \in A} B_\lambda$. Then B is a subsemigroup of M.
Since B_λ is a left tri-ideal of M, we have

$$\begin{aligned} &B_\lambda M B_\lambda B_\lambda \subseteq B_\lambda, \text{ for all } \lambda \in A \\ \Rightarrow &\cap B_\lambda M \cap B_\lambda \cap B_\lambda \subseteq \cap B_\lambda \\ \Rightarrow &BMBB \subseteq B. \end{aligned}$$

Hence B is a left tri-ideal of M. □

Theorem 3.17. *Let* B *be a left tri-ideal of asemigroup* M *,*$e \in B, eB \subseteq B$ *and* e *be* $\beta-$*idempotent. Then* eB *is a left tri-ideal of* M.

Proof. Let B be a left tri-ideal of the semigroup M. Suppose $x \in B \cap eM$. Then $x \in B$ and $x = ey, y \in M$.

$$\begin{aligned} x &= ey \\ &= eey \\ &= e(ey) \\ &= ex \in eB. \end{aligned}$$

$$\begin{aligned} \text{Therefore } &B \cap eM \subseteq eB \\ &eB \subseteq B \text{ and } eB \subseteq eM \\ \Rightarrow &eB \subseteq B \cap eM \\ \Rightarrow &eB = B \cap eM. \end{aligned}$$

Hence eB is a left tri-ideal of M. □

Corollary 3.15. *Let* M *be a semigroup* M *and* e *be idempotent. Then* eM *and* Me *are left tri-ideal and right tri-ideal of* M *respectively.*

Theorem 3.18. *Let* M *be a semigroup. If* $M = Ma$, *for all* $a \in M$. *Then every left tri-ideal of* M *is a quasi ideal of* M.

Proof. Let B be a left tri-ideal of the semigroup M and $a \in B$. Then

$$\begin{aligned} \Rightarrow &Ma \subseteq MB, \\ \Rightarrow &M \subseteq MB \subseteq M \\ \Rightarrow &MB = M \\ \Rightarrow &BM = BMB = BMBB \subseteq B \\ \Rightarrow &MB \cap BM \subseteq M \cap B \subseteq B. \end{aligned}$$

Therefore B is a quasi ideal of M. Hence the theorem. □

4. Left tri-simple semigroup

In this section,we introduce the notion of left tri-simple semigroup and characterize the left tri-simple semigroup using left tri- ideals of semigroup, study the properties of minimal left tri- ideals of a semigroup

DEFINITION 4.1. *A semigroup M is a left (right) simple semigroup if M has no proper left (right) ideals of M.*

DEFINITION 4.2. *A semigroup M is said to be simple semigroup if M has no proper ideals of M.*

DEFINITION 4.3. *A semigroup M is said to be bi- simple semigroup if M has no proper bi- ideals of M.*

DEFINITION 4.4. *A semigroup M is said to be left(right) tri- simple semigroup if M has no left(right) tri-ideal other than M itself.*

DEFINITION 4.5. *A semigroup M is said to be tri- simple semigroup if M has no tri-ideal other than M itself.*

THEOREM 4.1. *If M is a group then M is a tri- simple semigroup.*

PROOF. Let B be a proper left tri-ideal of the group M, $x \in M$ and $a \in B$. Since M is a group, there exist $b \in M$, such that $ab = 1$. Then $abx = x = xab$. Therefore $x \in BM$ and $M \subseteq BM$. We have $BM \subseteq M$. Hence $M = BM$. Similarly we can prove $MB = M$.

$$\begin{aligned} M &= MB = BMB = BMBB \\ &\subseteq B \\ M &\subseteq B \\ \text{Therefore, } M &= BM = BM = BBMB \\ &\subseteq B \\ M &\subseteq B \\ \text{Therefore, } M &= B. \end{aligned}$$

Hence group M has no proper -tri-ideals. □

THEOREM 4.2. *Let M be a left simple semigroup. Every left tri-ideal of M is a right ideal of M.*

PROOF. Let M be a left simple semigroup and B be a left tri-ideal of M. Then $BMBB \subseteq B$ and MB is a left ideal of M.
Since M is a left simple semigroup, we have $MB = M$. Therefore

$$\begin{aligned} BMBB &\subseteq B \\ \Rightarrow BM &\subseteq B. \end{aligned}$$

Hence the theorem. □

COROLLARY 4.1. *Let M be a right simple semigroup. Every right tri-ideal is a left ideal of M.*

Corollary 4.2. *Let M be a left and a right simple semigroup. Every tri-ideal is an ideal of M.*

Theorem 4.3. *Let M be a semigroup. M is a left tri-simple semigroup if and only if $< a >= M$, for all $a \in M$ and where $< a >$ is the smallest left tri-ideal generated by a.*

Proof. Let M be a semigroup. Suppose M is the left tri-simple semigroup, $a \in M$ and $B = Ma$.
Then B is a left ideal of M.
Therefore, by Theorem[3.5], B is a left tri-ideal of M.
Therefore $B = M$. Hence $Ma = M$, for all $a \in M$.

$$Ma \subseteq < a > \subseteq M$$
$$\Rightarrow M \subseteq < a > \subseteq M.$$
$$\text{Therefore } M = < a > .$$

Suppose $< a >$ is the smallest left tri-ideal of M generated by a, $< a >= M$, A is the left tri-ideal and $a \in A$. Then

$$< a > \subseteq A \subseteq M$$
$$\Rightarrow M \subseteq A \subseteq M.$$

Therefore $A = M$. Hence M is a left tri simple semigroup. □

Theorem 4.4. *If semigroup M is a left simple semigroup then every left tri-ideal of M is a right ideal of M.*

Proof. Let B be a left tri-ideal of the left simple semigroup$M$. Then MB is a left ideal of M and $MB \subseteq M$. Therefore $MB = M$. Then

$$\Rightarrow BMBB \subseteq B$$
$$\Rightarrow = BMB \subseteq B$$
$$\Rightarrow BM \subseteq B.$$

Hence every left tri-ideal is a right ideal of M.

□

Corollary 4.3. *Ifsemigroup M is right simple semigroup then every right tri-ideal of M is a left ideal of M.*

Corollary 4.4. *Every tri-ideal of left and right simple semigroup M is an ideal of M.*

Theorem 4.5. *Let M be a semigroup and B be a left tri-ideal of M. Then B is a minimal left tri-ideal of M if and only if B is a left tri- simple subsemigroup of M.*

Proof. Let B be a minimal left tri-ideal of the semigroup M and C be a left tri-ideal of B. Then $CBCC \subseteq C$.

and $CBCC$ is a left tri-ideal of M.
Since C is a tri-ideal of B,

$$\begin{aligned}&CBCC = B\\ \Rightarrow &B = CBCC \subseteq C\\ \Rightarrow &B = C.\end{aligned}$$

Conversely suppose that B is the left tri-simple subsemigroup of M. Let C be a left tri-ideal of M and $C \subseteq B$.

$\Rightarrow CBCC \subseteq CMCC \subseteq BMBB \subseteq B,$.Therefore C is a left tri-ideal of B.

$\Rightarrow B = C$.Since B is a left tri-simple subsemigroup of M.

Hence B is a minimal left tri-ideal of M. □

Corollary 4.5. *Let M be a semigroup and B be a right tri-ideal of M. Then B is a minimal right tri-ideal of M if and only if B is a right tri- simple subsemigroup of M.*

Corollary 4.6. *Let M be a semigroup and B be a tri-ideal of M. Then B is a minimal tri-ideal of M if and only if B is a tri- simple subsemigroup of M.*

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