BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS/BULLETIN
Bull. Int. Math. Virtual Inst., **14**(2)(2024), 225–232
DOI: 10.7251/BIMVI2402225S

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

SOME NOTES ON R-NEAR TOPOLOGICAL SPACES

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Abstract. In this study, the concepts of r-near base and r-near subspaces in r-near topological spaces defined by Atmaca using near approximation spaces are introduced. In addition, the basic features provided by these concepts are examined and examples are given.

1. Introduction

A set in the classical sense has a precise definition as to whether its element belongs to the set or not. However, recent scientific developments have revealed that many problems contain uncertainties rather than certainties. Therefore, problems involving uncertain data should have different mathematics than classical manner. Many theories have been put forward based on uncertainties. None of these theories are alternatives to classical mathematics. On the contrary, they contain many traditional mathematical concepts such as sets, functions, etc.

We try to establish nearness between objects, events or abstract concepts around us. Therefore, nearness is not a concept that can be expressed with precise definitions. For this reason, the near set theory, which is a new mathematical tool for such uncertainties, was defined by Peters in 2007. In this theory, the nearness between objects is established with the help of the relation called the indiscernibility relation which is obtained with the help of functions that representing object properties. Although this theory is a new theory, it has been applied in many fields of modern science ([**3**], [**4**], [**5**], [**6**], [**8**], [**9**], [**10**], [**11**], [**12**], [**13**]). One of these fields is topology, which is one of the important branches of mathematics. In 2020,

2020 *Mathematics Subject Classification.* Primary 54B05; Secondary 03E72.
Key words and phrases. Near set, r-near topology, topology.
Communicated by Dusko Bogdanic.

Atmaca [**1**] introduced the notion of r-near topological space and studied its fundamental properties. After this study, Atmaca and Zorlutuna [**2**] defined the concept of continuity on r-near topological spaces.

In this study, the r-near base and r-near subspaces, which are the continuation of the above mentioned studies, are introduced. In addition, the fundamental properties provided by these concepts have been studied and examples have been given.

2. Preliminaries

In this section, we give properties and some definitions of near sets defined by Peters [[**8**], [**9**]].

Definition 2.1. *[**8**]Let O be set of perceptual objects, F be a set of functions representing object features, $x, x' \in O$ and $B \subseteq F$. Then*
$\sim_B = \{(x, x') \in O \times O : \varphi(x) = \phi(x), \forall \varphi, \phi \in B\}$
is called the indiscernibility relation on O.

Definition 2.2. *[**8**]Let O be set of perceptual objects, F be a set of functions representing object features, $B \subseteq F$ and $\sim_B$be indiscernibility relation. Then the triple $FAS = (O, F, \sim_B)$ is called Fundamental Approximation Space.*

Definition 2.3. *[**8**]Let $(O, F, \sim_B)$ be a Fundamental Approximation Space and $A \subseteq O$.*

(1) The set of union of $[x]_B \in O/\sim_B$ which is subset of A, is called B lower approximation of A and denoted as
$B_*A = \underset{[x]_B \subseteq A}{\cup} [x]_B$.

(2) The set of union of $[x]_B \in O/\sim_B$ elements, whose intersection with A is non-empty, is called B upper approximation of A and defined as
$B^*A = \underset{[x]_B \cap A \neq \varnothing}{\cup} [x]_B$.

(3) The boundary of A is denoted as Bnd_BA and defined as
$Bnd_BA = B^*A \backslash B_*A = \{x \in O : x \in B^*A \text{ and } x \notin B_*A\}$.

Indiscernibility relation can be defined for each subset B_r, such that $B_r \subseteq B \subseteq F$ and $|B_r| = r$. Let us denote that relation with $\sim_{B_r}$. $\sim_{B_r}$can form different decomposition for each r over O. Here, $\sim_{B_r}$separate O to $[x]_{B_r}$ nearness classes and $O/\sim_{B_r} = \{[x]_{B_r} : x \in O\}$ set is quotient set. Consequently, $N_r(B) = \{O/\sim_{B_r} : B_r \subseteq B\}$ partition set is obtained.

Definition 2.4. *[**8**]Let O be a set of perceptual objects, F be a set of functions representing object features and $B \subseteq F$. Then quadruple of $NAS = (O, F, \sim_{B_r}, N_r(B))$ is called Nearness Approximation Space.*

Definition 2.5. *[**8**]Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space and $A \subseteq O$.*

(1) The union of $[x]_{B_r} \in O/\sim_{B_r}$ elements, whose are subset of A, is called B_r lower Approximation of A and defined as

$N_r(B)_*A = \underset{[x]_{B_r} \subseteq A}{\cup} [x]_{B_r}$.

(2) The union of $[x]_{B_r} \in O/\sim_{B_r}$ *elements, whose intersection with* A *is non-empty, is called* B_r *upper Approximation of* A *and defined as* A

$N_r(B)^*A = \underset{[x]_{B_r} \cap A \neq \varnothing}{\cup} [x]_{B_r}$.

DEFINITION 2.6. *[**1**]Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $X \subseteq O$ *and* (X, τ) *be a topological space. The family* $\tau_r^* = \{N_r(B)^*(G) : G \in \tau\}$ *is called* r*-near topology which generated by the family* $(O, F, \sim_{B_r}, N_r(B))$*. The elements* $N_r(B)^*(G)$ *are called* r*-near open sets. Complement of* r*-near open sets are called* r*-near closed sets.*

Throughout the paper, $N_r(B)^*(G)$ r*-near open sets are denoted as* G_r^* *for simplicity unless specified otherwise. The family of all* r*-near closed sets in* τ_r^* *is denoted as* τ_r^{*k}.

DEFINITION 2.7. *[**1**]Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $X \subseteq O$ *and* (X, τ) *be a topological space. If there exists a* $G_r^* \in \tau_r^*$ *such that* $x \in G \subseteq G_r^* \subseteq N$*, then* N *is called* r*-near neighborhood of* x*. If* N r*-near open set, then* N *is called* r*-near open neighborhood.*

The family of all r*-near neighborhood of* x *is denoted* $N_r^*(x)$ *and the family of all* r*-near open neighborhood is denoted as* $\tau_r^*(x)$.

DEFINITION 2.8. *[**1**]Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $X \subseteq O$*,* (X, τ) *be a topological space and* $A \subseteq X$.

(1) The set $cl_r^{*\tau}A = \cap\{F_r^* : F_r^*$ *is a* r*-near closed set and* $A \subseteq F_r^*\}$ *is called* r*-near closure of* A.

(2) The set $int_r^{*\tau}A = \cup\{G_r^* : G_r^*$ *is a* r*-near open set and* $G_r^* \subseteq A\}$ *is called* r*-near interior of* A.

DEFINITION 2.9. *[**2**]Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $X \subseteq O$*,* (X, τ) *and* (Y, σ) *be topological spaces and* $f : X \to Y$ *be a function. Then* f *is* r*-near continuous if and only if for all* $V \in N(f(x_0))$*, there exist* $U \in N_r^*(x_0)$ *such that* $f(U) \subseteq V$.

If f r*-near continuous of all* $x_0 \in X$*, then we say that* f *is* r*-near continuous.*

THEOREM 2.1. *[**2**]Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $X \subseteq O$*,* (X, τ) *and* (Y, σ) *topological spaces. The function* $f : X \to Y$ *is* r*-near continuous if and only if* U *is neighborhood of* $f(x_0)$*, then* $f^{-1}(U)$ *is* r*-near neighborhood of* x_0.

THEOREM 2.2. *[**2**]Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $X \subseteq O$*,* (X, τ) *and* (Y, σ) *be topological spaces and* $f : X \to Y$ *be a function. If* f *is* r*-near continuous, then* $f^{-1}(G) \in \tau_r^*$ *for all* $G \in \sigma$.

CONCLUSION 2.1. [**2**]Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space, $X \subseteq O$, (X, τ) and (Y, σ) be topological spaces and $f : X \to Y$ be a function. If $f^{-1}(G) \notin \tau_r^*$ for all $G \in \sigma$, then f is not r-near continuous.

THEOREM 2.3. *[**2**]Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $X \subseteq O$*,* (X, τ) *and* (Y, σ) *be topological spaces and* $f : X \to Y$ *be a function. If* f

is r-near continuous, then $f^{-1}(F)$ are r-near closed sets in τ_r^* for all F closed sets in σ.

3. r-near base

Definition 3.1. *Let $(O, F, \sim_{B_r}, N_r(B))$ be a nearness approximation space, $X \subseteq O$, (X, τ) be topological space and $\mathcal{D}$ be family of subsets of X. If each open set of τ_r^* different from the empty set can be written as a union of elements of $\mathcal{D}$, then $\mathcal{D}$ is called r-near base of τ_r^*.*

example 3.1. *Let $X = \{a, b, c, d\}$, $F = \{\varphi_1, \varphi_2, \varphi_3\}$ a set of the functions $\varphi_i : X \to \mathbb{R}$ which representing object features and defined as*

	a	b	c	d
φ_1	1	1	0	0
φ_2	0	0	1	1
φ_3	1	0	0	0

If we take the topology $\tau = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$and the families $\mathcal{D}_1 = \{X, \{a, b\}, \{b, c, d\}\}$, $\mathcal{D}_2 = \{\{a, b\}, \{c, d\}\}$, $\mathcal{D}_3 = \{\{a\}, \{b\}, \{c, d\}\}$. Then we have $\tau_1^* = \{\varnothing, X, \{a, b\}, \{b, c, d\}\}$, $\tau_2^* = \{\varnothing, X, \{a, b\}, \{c, d\}\}$ and $\tau_3^* = \{\varnothing, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clear that families $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ are r-near bases of τ_1^*, τ_2^* and τ_3^*, respectively.

Theorem 3.1. *Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space, $X \subseteq O$, (X, τ) be topological space and $\mathcal{D}$ be the base of τ. Then $\mathcal{D}_r^* = \{D_r^* \mid D \in \mathcal{D}\}$ families are the r-near base of the τ_r^* for all r.*

Proof. Let $G_r^* \in \tau_r^*$. Then

$y \in \bigcup_{D_r^* \in \mathcal{A}_r^*} D_r^*$, where $\mathcal{A}_r^* \subset \mathcal{D}_r^*$

$\iff y \in \bigcup_{[x]_{B_r} \cap D \neq \varnothing} [x]_{B_r}$ and $D_r^* \in \mathcal{A}_r^*$

$\iff y \in [x]_{B_r}$, for $D_r^* \in \mathcal{A}_r^*$, $[x]_{B_r} \cap D \neq \varnothing$

$\iff y \in [x]_{B_r}$, *for* $D \in \mathcal{A}$, $[x]_{B_r} \cap D \neq \varnothing$

$\iff y \in [x]_{B_r}$, *for* $[x]_{B_r} \cap (\bigcup_{[x]_{B_r} \cap D \neq \varnothing} D) \neq \varnothing$ and $D \in \mathcal{A}$

$\iff y \in \bigcup_{[x]_{B_r} \cap (\bigcup_{[x]_{B_r} \cap D \neq \varnothing} D) \neq \varnothing} [x]_{B_r}$

$\iff y \in \bigcup_{[x]_{B_r} \cap G \neq \varnothing} [x]_{B_r}$

$\iff y \in G_r^*$

This completes the proof. □

Theorem 3.2. *Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space, $X, Y \subseteq O$, (X, τ) and (Y, σ) topological spaces. If the function $f : X \to Y$ is r-near continuous, then $f^{-1}(D) \in \tau_r^*$ for all $D \in \mathcal{D}_r^*$, where $\mathcal{D}_r^*$ is r- near base of σ_r^*.*

Proof. Proof is obtained directly from the definition of continuity. □

THEOREM 3.3. *(A "Recognition Lemma") Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space, $X \subseteq O$, (X, τ) be topological space and $\mathcal{D} \subseteq \tau$. Then $\mathcal{D}_r^*$ is a base for τ_r^* if and only if there exist $D_r^* \in \mathcal{D}_r^*$ such that $x \in D_r^* \subset G_r^*$ for all $G_r^* \in \tau_r^*$ and $x \in G_r^*$.*

PROOF. ($\Longrightarrow$:) Let $\mathcal{D}_r^*$ be a base for τ_r^*, $G_r^* \in \tau_r^*$ and $x \in G_r^*$. Since $\mathcal{D}_r^*$ a base, it can be written in the form

$$G_r^* = \bigcup_{D_r^* \in \mathcal{A}_r^*} D_r^*$$

where $\mathcal{A}_r^* \subset \mathcal{D}_r^*$. Therefore

$$x \in G_r^*$$
$$\Longrightarrow x \in \bigcup_{D_r^* \in \mathcal{A}_r^*} D_r^*$$
$$\Longrightarrow x \in D_r^* \subset G_r^*, \textit{ for } D_r^* \in \mathcal{A}_r^*$$

($\Longleftarrow$:) Let $D_r^* \in \mathcal{D}_r^*$, where $x \in D_r^* \subset G_r^*$ for all $G_r^* \in \tau_r^*$ and $x \in G_r^*$. Then

$$G_r^* = \bigcup_{x \in G_r^*} \{x\} \subset \bigcup_{x \in D_r^* \subset G_r^*} D_r^* \subset G_r^*$$

Therefore, for all $G_r^* \in \tau_r^*$

$$G_r^* = \bigcup_{D_r^* \in \mathcal{A}_r^*} D_r^*$$

where $\mathcal{A}_r^* = \{(D_r^*)_x \mid x \in G_r^* \textit{ and } x \in (D_r^*)_x \subset G_r^*\}$. □

THEOREM 3.4. *Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space, $X \subseteq O$, and $\mathcal{D}_r^*$ the family of upper approximations of some subsets of X. If $\mathcal{D}_r^*$ is r-near base for a r-near topology on X, then X is written as a union of elements of $\mathcal{D}_r^*$.*

PROOF. Let $\mathcal{D}_r^*$ be base of a r-near topology τ_r^* on X. Then there exist $\mathcal{A}_r^* \subset \mathcal{D}_r^*$ such that

$$X = \bigcup_{D_r^* \in \mathcal{A}_r^*} D_r^*$$

Since $\mathcal{A}_r^* \subset \mathcal{D}_r^*$, $X = \bigcup_{D_r^* \in \mathcal{A}_r^*} D_r^* \subset \bigcup_{D_r^* \in \mathcal{D}_r^*} D_r^*$. Consequently, this shows that $X = \bigcup_{D_r^* \in \mathcal{D}_r^*} D_r^*$. □

4. r-near subspaces

DEFINITION 4.1. *Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space, $A \subseteq O$ and $F|_A = \{\varphi_i|_A : \varphi_i \in F$ and $i \in I\}$. Then $\sim_{B_r}^A = \{(x, x') \mid \varphi_i|_A(x) = \varphi_i|_A(x'), \varphi_i|_A \in B_r|_A \subset F|_A\}$ is equivalence relation and the equivalence classes are denoted by $[x]_{B_r}^A$.*

DEFINITION 4.2. *Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space, $F|_A = \{\varphi_i|_A : \varphi_i \in F$ and $i \in I\}$ and $A \subseteq O$. Then $(A, F|_A, \sim_{B_r}^A, N_r^A(B))$ is a Nearness Approximation Space and called sub-near approximation space.*

DEFINITION 4.3. *Let $(O, F, \sim_{B_r}, N_r(B))$ be a Nearness Approximation Space, (X, τ) topological space and $A \subseteq X \subseteq O$. The family $\tau_r^{*A} = \{G_r^{*A} : G_r^{*A} = \bigcup_{[x]_{B_r}^A \cap G_A \neq \varnothing} [x]_{B_r}^A, G_A \in \tau^A\}$ is called r-near subspace topology on A.*

EXAMPLE 4.1. *Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $X = \{a, b, c, d\}$, $F = \{\varphi_1, \varphi_2, \varphi_3\}$, $\tau = \{\varnothing, X, \{a\}, \{a, b\}, \{a, b, c\}\}$. *If we take* $A = \{b, c, d\}$ *and the function* φ_i *defined as*

	a	b	c	d
φ_1	1	0	0	1
φ_2	1	0	0	0
φ_3	1	1	0	0

. *Then the function* $\varphi_i|_A$ *defined as*

	b	c	d
$\varphi_1\|_A$	0	0	1
$\varphi_2\|_A$	0	0	0
$\varphi_3\|_A$	1	0	0

and $\tau^A = \{\varnothing, A, \{b\}, \{b, c\}\}$. *Therefore the set of equivalance classes for each* r*-combination are formed as*

$N_1^A(B) = \{\{[b]_{\{\varphi_1|_A\}} = \{b, c\},\ [d]_{\{\varphi_1|_A\}} = \{d\}\},\ \{[b]_{\{\varphi_2|_A\}} = \{b, c, d\}\},$ $\{[b]_{\{\varphi_3|_A\}} = \{b\},\ [c]_{\{\varphi_3|_A\}} = \{c, d\}\}\}$

$N_2^A(B) = \{\{[b]_{\{\varphi_1|_A, \varphi_2|_A\}} = \{b, c\},\ [d]_{\{\varphi_1|_A, \varphi_2|_A\}} = \{d\}\},\ \{[b]_{\{\varphi_1|_A, \varphi_3|_A\}} =$ $\{b\},\ [c]_{\{\varphi_1|_A, \varphi_3|_A\}} = \{c\},$
$[d]_{\{\varphi_1|_A, \varphi_3|_A\}} = \{d\}\},\ \{[b]_{\{\varphi_2|_A, \varphi_3|_A\}} = \{b\}\ , [c]_{\{\varphi_2|_A, \varphi_3|_A\}} = \{c, d\}\}\}$

$N_3^A(B) = \{\{[b]_{\{\varphi_1|_A, \varphi_2|_A, \varphi_3|_A\}} = \{b\},\ [c]_{\{\varphi_1|_A, \varphi_2|_A, \varphi_3|_A\}} = \{c\},$
$[d]_{\{\varphi_1|_A, \varphi_2|_A, \varphi_3|_A\}} = \{d\}\}\}$

Consequently, we have that $\tau_1^{*A} = \{\varnothing, A\}$, $\tau_2^{*A} = \{\varnothing, A, \{b, c\}\}$ *and* $\tau_3^{*A} =$ $\{\varnothing, A, \{b\}, \{b, c\}\}$.

PROPOSITION 4.1. *Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $F = \{\varphi_i \mid i \in I\}$ *and* $A \subseteq O$. *Then* $[x]_{B_r}^A = [x]_{B_r} \cap A$ *for the near approximation space* $(A, F|_A, \sim_{B_r}^A, N_r^A(B))$.

PROOF.
$y \in [x]_{B_r} \cap A$
$\Longleftrightarrow y \in [x]_{B_r}$ and $y \in A$
$\Longleftrightarrow x \sim_{B_r} y$ and $y \in A$
$\Longleftrightarrow \varphi_i(x) = \varphi_i(y)$ and $y \in A$, for all $\varphi_i \in B_r$
$\Longleftrightarrow \varphi_i|_A(x) = \varphi_i|_A(y)$, for all $\varphi_i|_A \in B_r|_A \subset F|_A$
$\Longleftrightarrow x \sim_{B_r}^A y$
$\Longleftrightarrow y \in [x]_{B_r}^A$ □

REMARK 4.1. If we consider the topologies τ_1^*, τ_2^* and τ_3^* in example 4.1, with the idea ofsubspace in classical manner the families $(\tau_1^*)_A = \{\varnothing, A, \{a, b\}\}$, $(\tau_2^*)_A =$ $\{\varnothing, A, \{a\}\}$, $(\tau_3^*)_A = \{\varnothing, A, \{a\}, \{a, b\}\}$ are obtained. But these families and r-near subspaces are different from each other. Let us give the following propositon shows relationship between r-near subspaces and these families.

THEOREM 4.1. *Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $A \subseteq X \subseteq O$ *and* (X, τ) *topological space. Then* $G_r^{*A} \subseteq (G_r^*)_A$ *for all* $G \in \tau$*, where* $(G_r^*)_A = G_r^* \cap A$.

PROOF. Let $G_r^{*A} \in \tau_r^{*A}$. Then

$$\begin{aligned} &G_r^{*A} \\ &= \bigcup_{[x]_{B_r}^A \cap G_A \neq \varnothing} [x]_{B_r}^A \\ &= \bigcup_{([x]_{B_r} \cap A) \cap (G \cap A) \neq \varnothing} ([x]_{B_r} \cap A) \\ &= (\bigcup_{[x]_{B_r} G \cap A \neq \varnothing} [x]_{B_r}) \cap A \\ &\subseteq (\bigcup_{[x]_{B_r} G \neq \varnothing} [x]_{B_r}) \cap A \\ &= (G_r^*) \cap A \end{aligned}$$

□

THEOREM 4.2. *Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $A \subseteq X \subseteq O$*,* (X, τ) *topological space, and* $F \subset A$*.* F_r^* *is r-near closed in* τ_r^{*A} *if and only if there exist* $K_r^* \in \tau_r^{*k}$ *such that* $F_r^* = A \cap K_r^*$*.*

PROOF. Let F be a subset of A. Then

F_r^* is closed in $(\tau_r^*)_A$

$\iff A \backslash F_r^* \in \tau_r^{*A}$

$\iff$ there exist $G_r^* \in \tau_r^*$ such that $A \backslash F_r^* = A \cap G_r^*$

$\iff$ there exist $G_r^* \in \tau_r^*$ such that $F_r^* = A \backslash (A \cap G_r^*) = A \cap (X \backslash G_r^*)$

$\iff F_r^* = A \cap K_r^*$, where $K_r^* = X \backslash G_r^*$ □

THEOREM 4.3. *Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* $A \subseteq X \subseteq O$ *and* (X, τ) *topological space.* $cl_r^{*\tau_A} E = A \cap cl_r^{*\tau} E$ *for* $E \subset A$*.*

PROOF. Let E be a subset of A. Then

$$\begin{aligned} &cl_r^{*\tau_A} E \\ &= \cap \{F_r^* : F_r^* \textit{ is closed in } \tau_r^{*A} \textit{ and } E \subset F_r^*\} \\ &= \cap \{A \cap K_r^* : K_r^* \textit{ is closed in } \tau_r^* \textit{ and } E \subset A \cap K_r^*\} \\ &= A \cap (\cap \{K_r^* : K_r^* \textit{ is closed in } \tau_r^* \textit{ and } E \subset K_r^*\}) \\ &= A \cap cl_r^{*\tau} E \end{aligned}$$

□

THEOREM 4.4. *Let* $(O, F, \sim_{B_r}, N_r(B))$ *be a Nearness Approximation Space,* (X, τ) *and* (Y, σ) *topological spaces,* $X, Y \subseteq O$*,* $A \subset X$*. If* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is continuous, then the function* $f|_A : A \longrightarrow Y$ *,defined as* $f|_A(x) = f(x)$*, is r-near continuous.*

PROOF. Let $x_0 \in A$ and $V \in N(f|_A(x_0))$. Then $V \in N(f(x_0))$ by definition of $f|_A$. Since f is r-near continuous, $f^{-1}(V) \in N_r^*(f(x_0))$ by Theorem 2.1 and so there exist $G \in \tau$ such that $x_0 \in G \subset G_r^* \subset f^{-1}(V)$. On the other hand, $G_A = \bigcup_{x \in G \cap A} \{x\} \subset \bigcup_{[x]_{B_r}^A \cap (G \cap A)} \{x\} = G_r^{*A}$. Then $x_0 \in (G \cap A) = G_A \subset G_r^{*A} \subset$

$f^{-1}(V) \cap A = f|_A(V)$. Therefore, $f|_A(V) \in N_r^{*\tau_A}(x_0)$ and this shows that $f|_A$ is r-near continuous.

□

Acknowledgements

This work is supported by the Scientific Research Project Fund of Sivas Cumhuriyet University under the project number F-610.

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Received by editors 27.6.2024; Revised version 15.9.2024; Available online 30.10.2024.

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