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On pairwise θ-generalized closed sets

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Abstract

The aim of this paper is to introduce a new notion of a pairwise generalized closed set called a pairwise θ-generalized closed set and study its fundamental basic properties. Furthermore, we introduced the concept of pairwise generalized Λ-sets which is extended to pairwise θ-generalized Λ-sets. Also we study some of their properties by using ij-θ-closure operator.

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1 Introduction and preliminaries

N. Levine ([10]) in 1970 introduced the notions of generalized closed sets and $T_{1/2}$ topological spaces. Recently, Noiri ([12]) gave another new generalization of Levine's g-closed set by utilizing the θ-closure operator. The concept of θ-generalized closed sets was applied to the digital line ([3]). This notion was studied extensively in the last few years by many topologists. In 1986, Fukutake ([4]) generalized this notion to bitopological spaces and he defined a set A of a bitopological space X to be an ij-generalized closed set (briefly ij-g-closed) if j-$cl(A) \subset U$ whenever $A \subset U$ and U is τ_i-open in X. Also he defined strongly pairwise $T_{1/2}$ which is stronger than that pairwise $T_{1/2}$ of Reilly ([13]).

In this paper we generalized the notion of ij-θ-generalized closed set by utilizing the ij-θ-closure operator defined in [1] and we defined θ-generalized continuity in bitopological spaces. Many basic properties of this new concept will be studied as well as its relations to the other classes of ij-generalized closed sets.

Throughout this paper (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or briefly X and Y) denote bitopological spaces. For a subset A of X, i-$cl(A)$ (resp. i-int(A)) denotes the

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closure of A and the interior of A respectively with respect to τ_i (or σ_i) for $i = 1, 2$. Also $i, j = 1, 2$ and $i \neq j$. A point x of A is said to be in the ij-θ-interior of A, denoted by ij-$\mathrm{int}_\theta(A)$ ([1]) if there exists $U \in \tau_i$ such that $x \in U \subset j$-$cl(U) \subset A$. A is said to be ij-θ-open ([1]) if $A = ij$-$\mathrm{int}_\theta(A)$.

The complement of an ij-θ-open set is called an ij-θ-closed (i.e. A is ij-θ-closed set if $A = ij$-$cl_\theta(A)$, where ij-$cl_\theta(A) = \{x \in X : j$-$cl(U) \cap A \neq \phi, U \in \tau_i$ and $x \in U\}$. The family of all ij-θ-open sets forms a topology $\tau_i(\theta)$ such that $\tau_i(\theta) \subset \tau_i$, for $i = 1, 2$ ([15]).

A subset A of X is said to be ij-regular open (resp. ij-regular closed) ([16]) if $A = i$-$\mathrm{int}(j$-$cl(A))$ (resp. $A = i$-$cl(j$-$\mathrm{int}(A)))$. A subset A of X is said to be ij-semi-open ([2]) (resp. ij-α-open ([7]), ij-semi-pre-open ([6]) if $A \subset j$-$cl(i$-$\mathrm{int}(A))$ (resp. $A \subset i$-$\mathrm{int}(j$-$cl(i$-$\mathrm{int}(A))), A \subset j$-$cl(i$-$\mathrm{int}(j$-$cl(A))))$.

The complement of an ij-semi-open (resp. ij-α-open, ij-semi-pre-open) is called ij-semi-closed (resp. ij-α-closed, ij-semi-pre-closed). The intersection of all ij-semi-closed (resp. ij-α-closed, ij-semi-pre-closed) is called ij-semi-closure (resp. ij-α-closure, ij-semi-pre-closure) of A and is denoted by ij-$scl(A)$ (resp. ij-$cl(A), ij$-$spcl(A))$.

Now, we mention the following definitions and results:

Definition 1.1 ([9]) *A subset A of a space X is called:*

(1) An ij-semi-generalized closed set (briefly ij-sg-closed) if ji-$scl(A) \subset U$ whenever $A \subset U$ and U is ij-semi-open in X.

(2) An ij-generalized α-closed set (briefly ij-gα-closed) if ji-$\alpha cl(A) \subset U$ whenever $A \subset U$ and U is ij-α-open in X.

(3) An ij-generalized semi-closed set (briefly ij-gs-closed) if ji-$scl(A) \subset U$ whenever $A \subset U$ and U is τ_i-open in X.

Definition 1.2 *A bitopological space (X, τ_1, τ_2) is called:*

(1) pairwise R_0-space (briefly PR_0-space) ([11]) *if for each τ_i-open U and $x \in U, \tau_j$-$cl(\{x\})U$.*

(2) pairwise R_1-space (briefly PR_1-space) ([14]) *if and only if for each $x, y \in X$ such that $x \notin \tau_i$-$cl(\{y\})$, there is a τ_i-open set U and a τ_j-open set V such that $x \in U, y \in V$ and $U \cap V = \phi$.*

(3) pairwise T_0 ([13]) *(briefly PT_0-space) if and only if for each $x, y \in X$ with $x \neq y$, there exists a τ_1-open set containing x but not y or a τ_2-open set containing y but not x.*

(4) pairwise $T_{1/2}$-space (briefly $PT_{1/2}$-space) ([4]) *if every ij-g-closed set is τ_j-closed.*

(5) pairwise Hausdorff ([5]) *if and only if for each $x, y \in X$ with $x \neq y$, there are a τ_i-open set U containing x and τ_j-open set V containing y such that $U \cap V = \phi$.*

(6) pairwise regular ([5]) *if for each $x \in X$ and each τ_i-closed F not containing x, there exist a τ_i-open set U and a τ_j-open set V such that $x \in U, F \subset V$ and $U \cap V = \phi$, equivalent for each τ_i-open set U and $x \in U$, there exists τ_i-open set V such that $x \in V \subset j\text{-}cl(V) \subset U$.*

(7) ij-extremely disconnected ([8]) *if $j\text{-}cl(U)$ is τ_i-open for every τ_i-open set U.*

Definition 1.3 *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called:*

(1) ij-g-continuous ([9]) if $f^{-1}(V)$ is ij-g-closed in X for every σ_j-closed set V of Y.

(2) ij-strongly θ-continuous [[1]] if for each $x \in X$ and each σ_i-open set V containing $f(x)$, there exists a τ_i-open set U containing x such that $f(j\text{-}cl(U)) \subset V$. If f is 12-strongly θ-continuous and 21-strongly θ-continuous, then f is called pairwise strongly θ-continuous.

Lemma 1.1 ([6]) *A subset A of a bitopological space X is ij-semi-pre-closed if and only if $A = ij\text{-}spcl(A)$.*

Lemma 1.2 ([15]) *Let A and B be a subsets of a bitopological space (X, τ_1, τ_2) then:*

(i) $A \subset B \Rightarrow ij\text{-}cl_\theta(A) \subset ij\text{-}cl_\theta(B)$.

(ii) $i\text{-}cl(A) \subset ij\text{-}cl_\theta(A)$.

(iii) $A \in \tau_j \Rightarrow i\text{-}cl(A) = ij\text{-}cl_\theta(A)$.

(iv) $ij\text{-}cl_\theta(A) = \cap\{i\text{-}cl(V) : A \subset V \in \tau_j\}$.

From Lemma 1.5 (iv), we note that $ij\text{-}cl_\theta(A)$ is a τ_i-closed set in X.

Lemma 1.3 ([4]) *A bitopological space (X, τ_1, τ_2) is an $P\text{-}T_{1/2}$-space if and only if for each $x \in X$, singleton $\{x\}$ is τ_j-open or τ_i-closed.*

Theorem 1.1 *A bitopological space (X, τ_1, τ_2) is be pairwise Hausdorff if and only if for each $x \in X$, $ij\text{-}cl_\theta(\{x\}) = \{x\}$.*

Proof. Let (X, τ_1, τ_2) be pairwise Hausdorff. Since for every singleton set $\{x\} \subset ij\text{-}cl_\theta(\{x\})$, then we show that $ij\text{-}cl_\theta(\{x\}) \subset \{x\}$. Let $y \notin \{x\}$ then there exist a τ_j-open set U containing x and τ_i-open set V containing y such that $U \cap V = \phi$ and $U \cap \tau_j\text{-}cl(V) = \phi$. Hence $\{x\} \cap \tau_j\text{-}cl(V) = \phi$ and $y \notin ij\text{-}cl_\theta(\{x\})$. Thus $ij\text{-}cl_\theta(\{x\}) \subset \{x\}$.

Conversely, let $ij\text{-}cl_\theta(\{x\}) = \{x\}$ and $x, y \in X$ such that $x \neq y$. Then $y \notin ij\text{-}cl_\theta(\{x\})$ and hence there exists τ_i-open set V containing y such that $j\text{-}cl(V) \cap \{x\} = \phi$. Thus $\{x\} \subset X \setminus j\text{-}cl(V) = U$ and U is a τ_j-open set containing x such that $U \cap V = \phi$. Hence (X, τ_1, τ_2) be pairwise Hausdorff. □

Theorem 1.2 ([11]) *A bitopological space (X, τ_1, τ_2) is pairwise R_0 if and only if each τ_j-closed F can be expressed as:*

$$F = \cap\{U : F \subset U \quad and \quad U is \tau_i\text{-}open\}$$

Lemma 1.4 *For a function $f : X \to Y$, the following are equivalent:*

(i) f is ij-strongly θ-continuous.

(ii) $f^{-1}(V)$ is ij-θ-open, for each σ_i-open set V of Y.

Proof. (i) ⇒(ii) Let V be σ_i-open set of Y and $x \in f^{-1}(V)$. Then there exists a τ_i-open U containing x such that $f(j\text{-}cl(U)) \subset V$ and hence $x \in U \subset j\text{-}cl(U) \subset f^{-1}(V)$. This shows that $f^{-1}(V)$ is ij-θ-open.
(ii)⇒(i) Let V be an ij-θ-open set containing $f(x)$. Then $x \in f^{-1}(V)$ and by (ii) there exists a τ_i-open set U containing x such that $x \in U \subset j\text{-}cl(U) \subset f^{-1}(V)$. Thus $f(j\text{-}cl(U)) \subset V$ and hence f is ij-strongly θ-continuous. □

2 Basic properties of pairwise θ-generalized closed sets

Definition 2.1 *A subset A of a space X is called:*

(1) An ij-α-generalized closed set (briefly ij-αg-closed) if ji-$\alpha cl(A) \subset U$ whenever $A \subset U$ and U is τ_i-open in X.

(2) An ij-generalized semi-preclosed set (briefly ij-gsp-closed) if ji-spcl$(A) \subset U$ whenever $A \subset U$ and U is τ_i-open in X.

(3) An ij-regular generalized closed set (briefly ij-r-g-closed) if j-cl$(A) \subset U$ whenever $A \subset U$ and U is ij-regular open in X.

Definition 2.2 *A subset A of a bitopological space (X, τ_1, τ_2) is called ij-θ-generalized closed (briefly ij-θ-g-closed) if ji-$cl_\theta(A) \subset U$ whenever $A \subset U$ and U is τ_i-open in X. If $A \subset X$ is 12-θ-g-closed and 21-θ-g-closed, then it said to be pairwise θ-generalized closed (briefly $P\theta$-g-closed).*

We denote the family of all ij-θ-generalized closed subsets of a space X by ij-$TGC(X)$.

Theorem 2.1 *Every ji-θ-closed set is ij-θ-g-closed.*

Proof. Let $A \subset X$ be ji-θ-closed and $A \subset U$ such that $U \in \tau_i$, then ji-$cl_\theta(A) = A$. Thus ji-$cl_\theta(A) \subset U$. Thus A is ij-θ-g-closed. □

The following example shows that the converse of Theorem 2.3 is not true in general.

Example 2.4. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{c\}, \{a, c\}, X\}$. The set $A = \{a, c\}$ is 12-θ-g-closed, since the only τ_1-open superset of A is X. But it is easy to see that A is not 21-θ-closed. In fact it is not even 21-semi-closed, since its complement is $\{b\}$ and τ_1-int$(\{b\}) = \phi$.

Theorem 2.2 *Let (X, τ_1, τ_2) be a bitopological space. Then:*

(i) Every ij-θ-g-closed set is ij-g-closed.

(ii) Every ij-θ-g-closed set is ij-αg-closed.

(iii) Every ij-θ-g-closed set is ij-gs-closed.

(iv) Every ij-θ-g-closed set is ij-r-g-closed.

Proof. Let A be a subset of X. By Lemma 1.5, $j\text{-}cl(A) \subset ji\text{-}cl_\theta(A)$. Thus if U is τ_i-open and $A \subset U$, then $j\text{-}cl(A) \subset ji\text{-}cl_\theta(A) \subset U$. Hence A is ij-g-closed. The proof of the other cases is similar. □

The next example shows that an ij-g-closed set, even a j-closed set, need not be always ij-θ-g-closed .

Example 2.6. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Let $A = \{c\}$, then A is τ_2-closed and hence 12-g-closed. If $U = \{a, c\} \in \tau_1$, 21-$cl_\theta(A) = X \not\subset U$. Thus, A is not 12-θ-g-closed.

Theorem 2.3 *Every ji-semi-pre-closed set is ij-gsp-closed.*

Proof. It follows from the fact that a set $A \subset X$ is ji-semi-preclosed if and only if $ji\text{-}spcl(A) = A$, by Lemma 1.4. □

Theorem 2.4 *Every ij-gs-closed set is ij-gsp-closed.*

Proof. Let A be ij-gs-closed. Since $ij\text{-}spcl(A) \subset ij\text{-}scl(A)$ for every subset A of X, then A is ij-gsp-closed . □

The reverse in the theorems above is not always true. Next we give an example of an ij-gsp-closed set which is not ij-gs-closed.

Example 2.9. Let $X = \{a, b, c, d, e\},\ \tau_1 = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ and $\tau_2 = \{\phi, \{a, c, e\}, \{b, d\}, X\}$. Let $A = \{b, c\}$ and $U = \{a, b, c, d\}$. Then 21-$spcl(A) = A$, so A is 21-semi-preclosed. Thus by Theorem 2.7, A is 12-gsp-closed. On the other hand $A \subset U \in \tau_1$ and 21-$scl(A) = X$.

This shows that A is not 12-gs-closed.

The next example shows that the converse of Theorem 2.7 is not true.

Example 2.10. Let $X = \{a, b, c\},\ \tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{a, b\}, \{c\}, X\}$. Let $A = \{a, c\}$. Clearly, A is not 21-semi-preclosed. On the other hand, A is

12-*gsp*-closed (even 12-*g*-closed and 12-*gs*-closed), since the only open set containing A is X.

Theorem 2.5 *Every ij-g-closed set is ij-r-g-closed.*

Proof. It is follows from the fact that every ij-regular open set of X is τ_i-open. □

From the above discussion and from the results in [9] we have the following diagram.

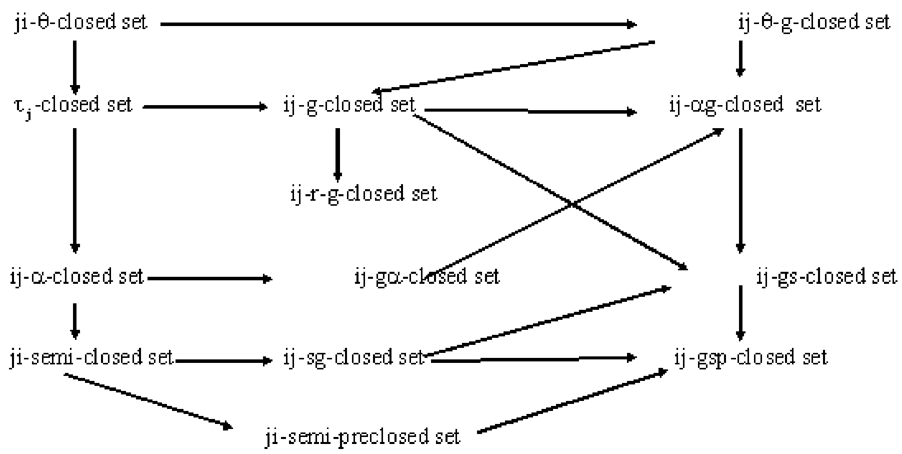

Figure 1:

Theorem 2.6 *A space X is pairwise regular if and only if $ij\text{-}cl_\theta(A) = i\text{-}cl(A)$, for each a subset A of X.*

Proof. Let (X, τ_1, τ_2) be a pairwise regular space. Since $i\text{-}cl(A) \subset ij\text{-}cl_\theta(A)$ we show that $ij\text{-}cl_\theta(A) \subset i\text{-}cl(A)$. Let $x \notin i\text{-}cl(A)$. Then there exists a τ_i-open set U and a τ_j-open set V such that $x \in U$, $A \subset V$ and $U \cap V = \phi$. Hence $j\text{-}cl(U) \cap A = \phi$ and $x \notin ij\text{-}cl_\theta(A)$. Thus $ij\text{-}cl_\theta(A) \subset i\text{-}cl(A)$.

Conversely, let $ij\text{-}cl_\theta(A) = i\text{-}cl(A)$ and F be a τ_i-closed set not containing x. Then $x \notin ij\text{-}cl_\theta(F)$ and hence there exists a τ_i-open set U containing x such that $j\text{-}cl(U) \cap F = \phi$. Thus, $F \subset X \setminus j\text{-}cl(U) = V$ and V is a τ_j-open set such that $U \cap V = \phi$. Hence, (X, τ_1, τ_2) is a pairwise regular space. □

Theorem 2.7 *Let (X, τ_1, τ_2) be a pairwise regular space. Then for a subset A of X the following are equivalent:*

(i) A is an ij-θ-g-closed set.

(ii) A is an ij-g-closed set.

Proof. (i) $\Rightarrow$ (ii) is valid for every bitopological space X by Theorem 2.5.

(ii)$\Rightarrow$(i) since a bitopological space X is ij-regular if and only if for each set $A \subset X, ij\text{-}cl_\theta(A) = i\text{-}cl(A)$, Theorem 2.12. Thus if $A \subset U$, where A is ij-g-closed and U is τ_i-open then $ji\text{-}cl_\theta(A) = j\text{-}cl(A) \subset U$. Hence, A is ij-θ-g-closed. *qed*

Lemma 2.1 *If A and B are subsets of a space X, then :*

(i) $ij\text{-}cl_\theta(A \cup B) = ij\text{-}cl_\theta(A) \cup ij\text{-}cl_\theta(B)$.

(ii) $ij\text{-}cl_\theta(A \cap B) \subset ij\text{-}cl_\theta(A) \cap ij\text{-}cl_\theta(B)$.

Proof. (i) First we prove that $ij\text{-}cl_\theta(A \cup B) \subset ij\text{-}cl_\theta(A) \cup ij\text{-}cl_\theta(B)$ for every subsets A and B of X. Let $x \notin ij\text{-}cl_\theta(A) \cup ij\text{-}cl_\theta(B)$, then there exist τ_i-open sets U and V containing x such that $j\text{-}cl(U) \cap A = \phi$ and $j\text{-}cl(V) \cap B = \phi$. This implies $x \in U \cup V$. Since $j\text{-}cl(U \cup V) = j\text{-}cl(U) \cup j\text{-}cl(V)$ holds, then we have $j\text{-}cl(U \cup V) \cap (A \cup B) = \phi$. Hence $x \notin ij\text{-}cl_\theta(A \cup B)$. Making use of Lemma 1.5, we have $ij\text{-}cl_\theta(A) \cup ij\text{-}cl_\theta(B) \subset ij\text{-}cl_\theta(A \cup B)$. This completes the proof.

(ii) Let $x \notin ij\text{-}cl_\theta(A) \cap ij\text{-}cl_\theta(B)$. Then there exists a τ_i-open set U containing x such that either $j\text{-}cl(U) \cap A = \phi$ or $j\text{-}cl(U) \cap B = \phi$. In any case we obtain that $j\text{-}cl(U) \cap (A \cap B) = \phi$. Thus $x \notin ij\text{-}cl_\theta(A \cap B)$. □

Theorem 2.8 *(i) A finite union of ij-θ-g-closed sets is always an ij-θ-g-closed set.*

(ii) A countable union of ij-θ-g-closed sets need not be an ij-θ-g-closed set.

(iii) A finite intersection of ij-θ-g-closed sets need not be an ij-θ-g-closed set.

Proof. (i) Let $A, B \subset X$ be ij-θ-g-closed sets. Let U be a τ_i-open subset of X such that $A \cup B \subset U$. By Lemma 2.14 (i), $ji\text{-}cl_\theta(A \cup B) = ji\text{-}cl_\theta(A) \cup ji\text{-}cl_\theta(B) \subset U \cup U = U$, since A and B are ij-θ-g-closed. Hence $A \cup B$ is ij-θ-g-closed.

(ii) Let X be the real line and $\tau_1 = \tau_2$, the usual topology on X. Since X is pairwise regular, then by Theorem 2.12, every singleton in X is ij-θ-g-closed. Let $A = \cup_{i=2}^{\infty}\{1/i\}$. Clearly A is a countable union of ij-θ-g-closed sets but A is not ij-θ-g-closed, since $A \subset (0, 1)$ and $0 \in ij\text{-}cl_\theta(A)$.

(iii) Let $X = \{a, b, c, d, e\}$, $\tau_1 = \{\phi, \{c\}, \{a, b\}, \{a, b, c\}, X\}$ and $\tau_2 = \{\phi, \{e\}, \{a, b, c\}, \{c, d, e\}, X\}$. Let $A = \{a, c, d\}$ and $B = \{b, c, e\}$. Clearly A and B are 12-θ-g-closed sets, since X is the only τ_1-open superset of them. But $C = \{c\} = A \cap B$ is not 12-θ-g-closed, since $C \subset \{c\} \in \tau_1$ and $21\text{-}cl_\theta(C) = \{c, d\} \not\subset \{c\}$. □

Definition 2.3 *A subset A of a space X whose τ_i-closure is τ_j-open is called ij-CO-set.*

Theorem 2.9 *If a subset A of a space X is an ij-θ-g-closed and ji-CO-set, then ji-$cl_\theta(A) = j$-$cl(A)$.*

Proof. Clearly, j-$cl(A) \subset ji$-$cl_\theta(A)$. We show that ji-$cl_\theta(A) \subset j$-$cl(A)$. Since A is a ji-CO-set, then j-$cl(A)$ is τ_i-open. Since A is a ij-θ-g-closed and $A \subset j$-$cl(A)$, then ji-$cl_\theta(A) \subset j$-$cl(A)$. Thus ji-$cl_\theta(A) = j$-$cl(A)$. □

A subset A of a space X which is both i-closed and j-open is called ij-clopen ([8]).

Theorem 2.10 *For a subset A of a bitopological space (X, τ_1, τ_2) the following conditions are equivalent:*
(i) A is ji-clopen.
(ii) A is ij-θ-g-closed, ji-CO-set and ji-semi-closed.

Proof. (i) $\Rightarrow$ (ii) : Since A is j-closed, then it is ji-semi-open. By Definition 2.16, A is ji-clopen, then A is ji-CO-set. Now, let $A \subset U$ where $U \in \tau_i$. By Lemma 1.5, $A \in \tau_i$ implies that ji-$cl_\theta(A) = j$-$cl(A) \subset U$. Hence A is ij-θ-g-closed.

(ii)$\Rightarrow$(i) By Theorem 2.17, ji-$cl_\theta(A) = j$-$cl(A) = i$-int$(j$-$cl(A)) \subset A$, since A is a ji-semi-closed, ji-CO-set. Then $A = ji$-$cl_\theta(A)$, thus A is ji-θ-closed and τ_j-closed. Since $A = i$-int$(j$-$cl(A))$, then A is ij-regular open and hence A is τ_i-open. Then A is ji-clopen. □

3 Characterizations of $PT_{1/2}$-spaces, PT_1-spaces and PR_0-spaces

Theorem 3.1 *A bitopological space (X, τ_1, τ_2) is a P- $T_{1/2}$-space if and only if every ij-θ-g-closed set is τ_j-closed.*

Proof. Let $A \subset X$ be ij-θ-g-closed. By Theorem 2.5, A is ij-g-closed. Since X is a P-$T_{1/2}$-space, then A is τ_j-closed.

Conversely, let $x \in X$. If $\{x\}$ is not τ_i-closed, then $B = X \setminus \{x\}$ is not τ_i-open and thus X is the only τ_i-open superset of B. Hence B is ij-θ-g-closed. By assumption B is τ_j-closed or equivalently, $\{x\}$ is τ_j-open. Thus every singleton in X is τ_j-open or τ_i-closed. Hence by Lemma 1.6, X is a P-$T_{1/2}$-space. □

Lemma 3.1 *Let a subset A of a space X be ij-θ-g-closed. Then ji-$cl_\theta(A) \setminus A$ does not contain a non-empty τ_i-closed set.*

Proof. Let F be a τ_i-closed subset of $ji\text{-}cl_\theta(A) \setminus A$. Clearly, $A \subset X \setminus F$, where A is ij-θ-g-closed and $X \setminus F$ is τ_i-open. Thus $ji\text{-}cl_\theta(A) \subset X \setminus F$ or equivalently $F \subset X \setminus ji\text{-}cl_\theta(A)$. By assumption we have that $F \subset ji\text{-}cl_\theta(A)$, and thus $F \subset (X \setminus ji\text{-}cl_\theta(A)) \cap (ji\text{-}cl_\theta(A)) = \phi$. This shows that F is empty. □

Lemma 3.2 *In any bitopological space (X, τ_1, τ_2) a singleton is ij-θ-open if and only if it is ji-clopen.*

Proof. The assertion follows from the fact that a set A is ij-θ-open if and only if for each $x \in A$ there exist a τ_i-open set U and a τ_j-closed set V such that $x \in U \subset V \subset A$. □

Theorem 3.2 *A bitopological space (X, τ_1, τ_2) is a PT_1-space if and only if every ij-θ-g-closed set is ji-θ-closed.*

Proof. Let $A \subset X$ be ij-θ-g-closed and let $x \in ji\text{-}cl_\theta(A)$. Since X is PT_1, then $\{x\}$ is τ_i-closed and thus by Lemma 3.2 $x \notin ji\text{-}cl_\theta(A) \setminus A$. Since $x \in ji\text{-}cl_\theta(A)$, then $x \in A$. This shows that $ji\text{-}cl_\theta(A) \subset A$ or equivalently that A is ji-θ-closed.

Conversely, let $x \in X$ and assume that $\{x\}$ is τ_i-closed. Then $B = X \setminus \{x\}$ is not τ_i-open and clearly B is ij-θ-g-closed, since the only τ_i-open superset of B is X itself. By assumption B is ji-θ-closed and thus $\{x\}$ is ji-θ-open. By Lemma 3.3, $\{x\}$ is ij-clopen. Thus X is a PT_1-space. □

Definition 3.1 *A subset A of a bitopological space (X, τ_1, τ_2) is called a Λ_i-set if $A = A^{\Lambda_i}$, where $A^{\Lambda_i} = \cap\{U : A \subset U,\ U$ is τ_i-open$\}$.*

Definition 3.2 *A subset A of a bitopological space (X, τ_1, τ_2) is called a generalized Λ_{ij}-set (briefly g-Λ_{ij}-set) if $A^{\Lambda_i} \subset F$, whenever $A \subset F$ and F is τ_j-closed.*

Definition 3.3 *For a subset A of a bitopological space (X, τ_1, τ_2) we define $A_\theta^{\Lambda_{ij}}$ as follows $A_\theta^{\Lambda_{ij}} = \{x \in X : ij\text{-}cl_\theta\{x\} \cap A \neq \phi\}$.*

Lemma 3.3 *For any set $A \subset X$, $A \subset A^{\Lambda_i} \subset A_\theta^{\Lambda_{ij}} \subset ji\text{-}cl_\theta(A)$.*

Proof. Let A be a subset of a space X. Obviously, $A \subset A^{\Lambda_i}$. Now, we prove $A^{\Lambda_i} \subset A_\theta^{\Lambda_{ij}}$. Let $x \notin A_\theta^{\Lambda_{ij}}$. It follows that $ij\text{-}cl_\theta(\{x\}) \cap A = \phi$ and $A \subset X \setminus ij\text{-}cl_\theta(\{x\}) = U$. Since $ij\text{-}cl_\theta(\{x\})$ is ij-θ-closed, then U is ij-θ-open. Hence there exists a τ_i-open set U containing A but not x, then $x \notin A^{\Lambda_i}$. Thus $A^{\Lambda_i} \subset A_\theta^{\Lambda_{ij}}$.

Now we prove that $A_\theta^{\Lambda_{ij}} \subset ji\text{-}cl_\theta(A)$. Suppose that $x \notin ji\text{-}cl_\theta(A)$. Then there exists a τ_j-open set U such that $x \in U$ and $i\text{-}cl(U) \cap A = \phi$. Since U is τ_j-open, it follows by Lemma 1.5 $i\text{-}cl(U) = ij\text{-}cl_\theta(U)$. So $ij\text{-}cl_\theta(U) \cap A = \phi$. Thus, $ij\text{-}cl_\theta(\{x\}) \cap A = \phi$ and $x \notin A_\theta^{\Lambda_{ij}}$. Hence, $A_\theta^{\Lambda_{ij}} \subset ji\text{-}cl_\theta(A)$. □

Theorem 3.3 *A bitopological space (X, τ_1, τ_2) is a PT_1-space if and only if every subset in X is a Λ_i-set.*

Proof. Let U be a subset of a PT_1-space. Suppose that there exists a point $x \in X$ such that $x \notin U$. Then $X \setminus \{x\}$ is a τ_i-open set containing U. Thus $x \notin U^{\Lambda_i}$ and $U^{\Lambda_i} \subseteq U$. Since $U \subset U^{\Lambda_i}$, then $U = U^{\Lambda_i}$. Hence, U is Λ_i-set.

Conversely, assume that every set in X is a Λ_i-set. Let a and b be two different points, then $\{a\}^{\Lambda_i} \cap \{b\}^{\Lambda_j} = \phi$. Since $a \in \{a\}^{\Lambda_i}$, then $a \in \{b\}^{\Lambda_j}$ and so there exists a τ_j-open *nbd.* of b which does not contain a. On the other hand, since $b \in \{b\}^{\Lambda_j}$ then $b \notin \{a\}^{\Lambda_i}$ and so there exists a τ_i-open *nbd* of a which does not contain b. Thus X is a PT_1-space. □

Theorem 3.4 *Every ij-θ-closed set is a Λ_j-set.*

Proof. Let $A \subset X$ be ij-θ-closed. Then $A = ij\text{-}cl_\theta(A)$. Thus, by Lemma 3.8, $A^{\Lambda_j} \subset ij\text{-}cl_\theta(A) = A$ and hence $A = A^{\Lambda_j}$. Thus, A is a Λ_j-set. □

The following example shows that a Λ_j-set need not be ij-θ-closed.

Example 3.11. Let R be the set of real numbers and a space $X = R$, τ_1 be the co-finite topology on R, i.e., $\tau_1 = \{U \subseteq X : X \setminus U \text{ is finite}\} \cup \{\phi\}$ and τ_2 be the usual topology on R. Then (R, τ_1, τ_2) is a PT_1-space and any singleton of R is a Λ_j -set, but none of the singletons is ij-θ-closed.

Lemma 3.4 *If a subset A of a bitopological space (X, τ_1, τ_2) is ij-g-closed, then $j\text{-}cl(A) \subset A^{\Lambda_i}$.*

Proof. Let $A \subset X$ be ij-g-closed and $x \notin A^{\Lambda_i}$. Then there exists a τ_i-open set U of X containing x such that $x \notin U$ and $A \subset U$. By hypothesis, $j\text{-}cl(A) \subset U$. Hence, $x \notin j\text{-}cl(A)$ and $j\text{-}cl(A) \subset A^{\Lambda_i}$. □

Theorem 3.5 *Every ij-g-closed, Λ_i-set is τ_j-closed.*

Proof. Let $A \subset X$ be an ij-g-closed, Λ_i-set. Since A is ij-g-closed, then by Lemma 3.12, $j\text{-}cl(A) \subset A^{\Lambda_i}$. Moreover, since A is a Λ_i-set, then $j\text{-}cl(A) \subset A^{\Lambda_i} = A$. Thus, A is τ_j-closed. □

Definition 3.4 *A subset A of (X, τ_1, τ_2) is called θ-generalized-Λ_{ij}-set (briefly θ-g-Λ_{ij} -set) if $A_\theta^{\Lambda_{ij}} \subset F$ whenever $A \subset F$ and F is τ_j-closed in X.*

Theorem 3.6 *Every θ-g- Λ_{ij}-set is a g-Λ_{ij}-set.*

Proof. Let $A \subset X$ be a θ-g-Λ_{ij}-set and let $A \subset F$, where F is τ_j-closed. By assumption $A_\theta^{\Lambda_{ij}} \subset F$ and by Lemma 3.8 $A^{\Lambda_i} \subset A_\theta^{\Lambda_{ij}} \subset F$. This shows that A is a g-Λ_{ij}-set. □

Remark 3.16. A g-Λ_{ij}-set need not θ-g-Λ_{ij}-set. In Example 3.11, every singleton is a Λ_i-set and hence a g-Λ_{ij}-set but none of the singletons is a θ-g-Λ_{ij}-set, since the ji-θ-closure of every singleton is X.

Theorem 3.7 *A bitopological space (X, τ_1, τ_2) is a PR_0-space if and only if every singleton of X is a g-Λ_{ij}-set.*

Proof. In Theorem 1.8 shows that a space is PR_0 if and only if for each τ_j-closed set $A, A = A^{\Lambda_i}$. Thus if X is PR_0, then for each singleton $\{x\}$ and each τ_j-closed F containing x, we have $\{x\} \subset \{x\}^{\Lambda_i} \subset F^{\Lambda_i} = F$. So, $\{x\}$ is a g-Λ_{ij}-set.

Conversely, assume that $F \subset X$ is τ_j-closed. For each $x \in F$ by assumption $\{x\}^{\Lambda_i} \subset F$. Thus $F^{\Lambda_i} = \cup_{x \in F}\{x\}^{\Lambda_i} \subset F$. This shows that $F = F^{\Lambda_i}$ and X is a PR_0-space. □

Theorem 3.8 *Let (X, τ_1, τ_2) be a pairwise Hausdorff space. Then every subset of X is a θ-g-Λ_{ij}-set.*

Proof. By Theorem 1.7, every singleton in X is ij-θ-closed. Let $A \subset X$, it is easily observed that $A = A_\theta^{\Lambda_{ij}}$. Since, if $x \in A_\theta^{\Lambda_{ij}}$, then ij-$cl_\theta(\{x\} \cap A \neq \phi$ and $\{x\} \cap A \neq \phi$ or $x \in A$. Hence A is a θ-g-Λ_{ij}-set.

Theorem 3.9 *Let (X, τ_1, τ_2) be a bitopological space. Then the following statements are equivalent :*

(i) X is a pairwise R_1-space.

(ii) For each $x \in X, i$-$cl(\{x\})$ =ij–$cl_\theta(\{x\})$.

(iii) For each j-compact $A \subset X, i$-$cl(A) = ij$-$cl_\theta(A)$.

Proof. (i)⇒(iii): Generally i-$cl(A) \subset ij$-$cl_\theta(A)$. Now, let $x \notin i$-$cl(A)$. For each $y \in A$, $x \notin i$-$cl(\{y\})$ and so there exists a τ_i-open set U_y and a τ_j-open set V_y such that $x \in U_y, y \in V_y$ and $U_y \cap V_y = \phi$. Then $\{V_y : y \in A\}$ is a j-open cover of A. Since A is τ_j-compact, there exists a finite subset A_0 of A such that $A \subset \cup\{V_y : y \in A_0\}$. Put $V = \cup\{V_y : y \in A_0\}$ and $U = \cap\{U_y : y \in A_0\}$. Then V is a τ_j-open set, U is a τ_i-open set, $A \subset V, x \in U$ and $U \cap V = \phi$. Thus j-$cl(U) \cap V = \phi$ and so j-$cl(U) \cap A = \phi$. This shows that $x \notin ij$-$cl_\theta(A)$ and therefore ij-$cl_\theta(A) \subset i$-$cl(A)$.

(iii)⇒(ii): The proof is obvious, since $\{x\}$ is τ_j-compact.

(ii)⇒(i): Let $x \notin i$-$cl(\{y\})$, by (ii) $x \notin ij$-$cl_\theta(\{y\})$. Then there exists a τ_i -open set U such that $x \in U$ and j-$cl(U) \cap \{y\} = \phi$. Then $X \setminus j$-$cl(U)$ is a j-open set containing y. Also, $U \cap X \setminus j$-$cl(U) = \phi$. This shows that X is a pairwise R_1-space. □

4 On ij-θ-g-continuous and ij-θ-g-irresolute functions

Definition 4.1 *A function $f : (X, \tau_1, \tau_2) \to (y, \sigma_1, \sigma_2)$ is called is called:*

(1) ij-θ-g-continuous if $f^{-1}(V)$ is ij-θ-g-closed in X for every σ_j-closed set V of Y.

(2) ij-θ-g-irresolute if $f^{-1}(V)$ is ij-θ-g-closed in X for every ij-θ-g-closed set V of Y.

Theorem 4.1 *If a function $f : X \to Y$ is ji-strongly θ-continuous, then f is ij-θ-g-continuous.*

Proof. Let V be a σ_j-closed set of Y. Since f is ji-strongly θ-continuous, then by Lemma 1.9, $f^{-1}(V)$ is ji-θ-closed. By Theorem 2.3, $f^{-1}(V)$ is ij-θ-g-closed and hence f is ij-θ-g-continuous.

The following example shows that an ij-θ-g-continuous function need not be ji-strongly θ-continuous, not even ji-semi-continuous.

Example 4.3 Let $X = Y = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}, \tau_2 = \{\phi, \{a\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{b\}, \{a, b\}, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Clearly f is 12-θ-g-continuous but f is not 21-strongly θ-continuous, not even 21-semi-continuous.

Theorem 4.2 *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be ij-θ-g-continuous. Then f is ij-g-continuous.*

Proof. Let V be a σ_j-closed set of Y. Then $f^{-1}(V)$ is ij-θ-g-closed in X. By Theorem 2.5, every ij-θ-g-closed set is ij-g-closed. Then $f^{-1}(V)$ is ij-g-closed in X. Thus f is ij-g-continuous. □

The converse of the above theorem is not true in general and this can easily be seen from the following example.

Example 4.5. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$, $\tau_2 = \{\phi, \{c\}, \{a, b\}, X\}$, $\sigma_1 = \{\phi, \{a, b\}, Y\}$ and $\sigma_2 = \{\phi, \{c\}, Y\}$. Let $f : (X, \tau_1, \tau_2) \to (X, \sigma_1, \sigma_2)$ be the identity function. Clearly f is 2-continuous and hence 12-g-continuous but $A = \{c\}$ is not 12-θ-g-closed set in (X, τ_1, τ_2) and hence f is not 12-θ-g-continuous.

Example 4.3 and Example 4.5 also, show that i-continuity and ij-θ-g-continuity are independent concepts. Thus we have the following implications and none of them is reversible:

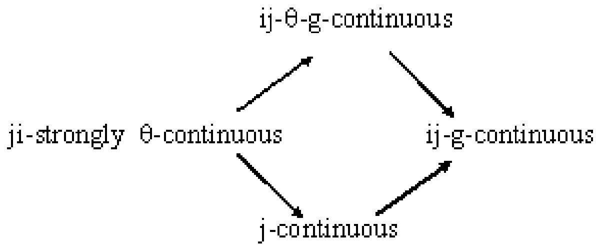

Figure 2:

Theorem 4.3 *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *and* $g : (Y, \sigma_1, \sigma_2) \to (Z, v_1, v_2)$ *be two functions. Then:*

(i) If g *is* j*-continuous and* f *is* ij*-*θ*-*g*-continuous, then* $g \circ f$ *is* ij*-*θ*-*g*-continuous.*

(ii) If g *is* ij*-*θ*-*g*-irresolute and* f *is* ij*-*θ*-*g*-irresolute, then* $g \circ f$ *is* ij*-*θ*-*g*-irresolute.*

(iii) If g *is* ij*-*θ*-*g*-continuous and* f *is* ij*-*θ*-*g*-irresolute, then* $g \circ f$ *is* ij*-*θ*-*g*-continuous.*

(iv) Let (Y, σ_1, σ_2) *be a pairwise regular space,* g *is* ij*-*g*-continuous and* f *is* ij*-*θ*-*g*-irresolute. Then* $g \circ f$ *is* ij*-*θ*-*g*-continuous.*

Proof.

(i) Let W be a v_j-closed set of Z. Since g is j-continuous, then $g^{-1}(W)$ is σ_j-closed set of Y. Moreover, f is ij-θ-g-continuous, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is ij-θ-g-closed set of X. Hence $g \circ f$ is ij-θ-g- continuous.

(ii) Let W be an ij-θ-g-closed set of Z. Since g is ij-θ-g-irresolute, then $g^{-1}(W)$ is ij-θ-g-closed set of Y. Since, f is ij-θ-g-irresolute, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is ij-θ-g-closed set of X. Hence $g \circ f$ is ij-θ-g-irresolute.

(iii) Let W be a v_j-closed set of Z. Since g is ij-θ-g-continuous, then $g^{-1}(W)$ is ij-θ-g-closed set of Y. Moreover, f is ij-θ-g-irresolute, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is ij-θ-g-closed set of X. Hence $g \circ f$ is ij-θ-g-continuous.

(iv) Let W be a v_j-closed set of Z. Since g is ij-g-continuous, then $g^{-1}(W)$ is ij-g-closed set of Y. Moreover, Y is pairwise regular, by Theorem 2.13, $g^{-1}(W)$ is ij-θ-g-closed and hence f ij-θ-g-irresolute, so $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is ij-θ-g-closed set of X. Thus $g \circ f$ is ij-θ-g-continuous. □

The following example shows that in general, the composition of two ij-θ-g-continuous functions is not ij-θ-g-continuous.

Example 4.7. Let f be the function in Example 4.3. Let $v_1 = v_2 = \{\phi, \{c\}, X\}$. Let $g : (X, \sigma_1, \sigma_2) \to (X, v_1, v_2)$ be the identity function. It is easily observed that g is also 12-θ-g-continuous. But the composition function $g \circ f : (X, \tau_1, \tau_2) \to (X, v_1, v_2)$ is not 12-θ-g-continuous, since $\{a, b\} \notin$ 12-$TGC(X, \tau_1, \tau_2)$.

Theorem 4.4 *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is bijective, i-open and ij-θ-g-continuous, then f is ij-θ-g-irresolute.*

Proof. Let V be ij-θ-g-closed set of Y and let $f^{-1}(V) \subset U$, where U be τ_i-open set. Clearly $V \subset f(U)$. Since $f(U) \in \sigma_i$ and since V is ij-θ-g-closed set in Y, then $ji\text{-}cl_\theta(V) \subset f(U)$ and thus $f^{-1}(ji\text{-}cl_\theta(V)) \subset U$. Since f is ij-θ-g-continuous and since $ji\text{-}cl_\theta(V)$ is σ_j-closed in Y, then $ji\text{-}cl_\theta(f^{-1}(ji\text{-}cl_\theta(V))) \subset U$ and hence $ji\text{-}cl_\theta(f^{-1}(V)) \subset U$. Therefore, $f^{-1}(V)$ is ij-θ-g-closed of X. Hence f is ij-θ-g-irresolute. □

Definition 4.2 *A subset A of bitopological space (X, τ_1, τ_2) is called ij-θ-generalized open (briefly ij-θ-g-open) if its complement $X \setminus A$ is ij-θ-g-closed in (X, τ_1, τ_2).*

Theorem 4.5 *(i) A subset A of (X, τ_1, τ_2) is ij-θ-g-open if and only if $F \subset ji\text{-}int_\theta(A)$, whenever $F \subset A$ and F is τ_i-closed in X.*

(ii) If A is ij-θ-g-open in (X, τ_1, τ_2) and B is ij-θ-g-open in (Y, σ_1, σ_2), then $A \times B$ is ij-θ-g-open in the product space $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$.

Proof. (i) Let $F \subset A$ and F be τ_i-closed in (X, τ_1, τ_2). Since $X \setminus A$ is ij-θ-g-closed and $X \setminus F$ is a τ_i-open set containing $X \setminus A$, we have $ji\text{-}cl_\theta(X \setminus A) = X \setminus ji\text{-}\mathrm{int}_\theta(A) \subset X \setminus F$. Therefore, $F \subset ji\text{-}\mathrm{int}_\theta(A)$.

Conversely, let U be a τ_i-open set of (X, τ_1, τ_2) such that $X \setminus A \subset U$. Then $X \setminus U$ is τ_i-closed and $X \setminus U \subset A$. Using assumption, $X \setminus U \subset ji\text{-}\mathrm{int}_\theta(A)$ and hence $ji\text{-}cl_\theta(X \setminus A) \subset U$. Thus A is ij-θ-g-open.

(ii) Let F be an i-closed subset of $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ such that $F \subset A \times B$, for each $(x, y) \in F, \tau_i\text{-}cl(\{x\}) \times \sigma_i\text{-}cl(\{y\}) \subset i\text{-}cl(F) = F \subset A \times B$. Then the two closed sets $\tau_i\text{-}cl(\{x\})$ and $\sigma_i\text{-}cl(\{y\})$ are contained in A and B respectively. By assumption, $\tau_i\text{-}cl(\{x\}) \subset ji\text{-}\mathrm{int}_\theta(A)$ and $\sigma_i\text{-}cl(\{y\}) \subset ij\text{-}\mathrm{int}_\theta(B)$ hold. This implies that for each $(x, y) \in F(x, y) \in ji\text{-}\mathrm{int}_\theta(A) \times ji\text{-}\mathrm{int}_\theta(B) \subset ji\text{-}\mathrm{int}_\theta(A \times B)$ and hence $F \subset ji\text{-}\mathrm{int}_\theta(A \times B)$. By (i) it is shown that $A \times B$ is ij-θ-g-open. □

Theorem 4.6 *The projection $p : (X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2) \to (X, \tau_1, \tau_2)$ is an ij-θ-g-irresolute map.*

Proof. By Definition 4.9 and Theorem 4.10(ii), for an ij-θ-g-closed set F of (X, τ_1, τ_2), $p^{-1}(X \setminus F) = (X \setminus F) \times Y$ is ij-θ-g-open in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$. Therefore, $p^{-1}(F) = F \times Y = X \times Y \setminus (p^{-1}(X \setminus F))$ is ij-θ-g-closed. □

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