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SOME TOPOLOGICAL PROPERTIES IN δ-HOMOGENEOUS NEUTROSOPHIC MODULAR SPACES

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Abstract. This paper introduces the concept of neutrosophic modular space. Afterward, a Hausdorff topology induced by a δ-homogeneous neutrosophic modular is defined and some related topological properties are also examined. After giving the fundamental definitions and the necessary examples, we introduce the definitions of Neutrosophic boundedness, neutrosophic compactness and neutrosophic convergence, and obtain several preservation properties and some characterizations concerning them. Also, we investigate the relationship between a neutrosophic modular and a neutrosophic metric. Finally, we prove some known results of metric spaces including Baire's theorem and the Uniform limit theorem for neutrosophic modular spaces.

1. Introduction

The notion of fuzzy sets was introduced by Zadeh [26] in 1965 and there are many viewpoints on the notion of metric space in topology. Kramosil and Michalek [13] introduced the concept of a metric space, which can be regarded as a generalization of the probabilistic metric space. Afterward, Grabiec [5] defined the metric spaces completeness and extended the Banach contraction theorem to the complete metric spaces. Next, George and Veeramani [6] modified the definition of the Cauchy sequence introduced by Grabiec. Atanassov [1] gave the concept of an intuitionistic set as a generalization of a set. Park [19] introduced the notion of an intuitionistic metric space as a natural generalization of a metric space due to

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George and Veeramani. He proved Baire's theorem and the uniform limit theorem for these spaces. For more details on intuitionistic metric space and related results, we refer the reader to [2, 18].

The concept of a modular space was founded by Nakano [16] and developed by Luxemburg [14]. Then, Musielak and Orlicz [15] redefined and generalized the notion of modular space Kozlowski [10, 11] introduced a modular function space. In the sequel, Kozlowski and Lewicki [12] considered the problem of analytic extension of measurable functions in modular function spaces and discussed some extension properties by means of polynomial approximation. Afterward, Kilmer and Kozlowski [8] studied the existence of best approximations in modular function spaces by elements of sub lattices. Nourouzi [17] proposed probabilistic modular spaces based on the theory of modular spaces and in [18] he extended the well-known Baire's theorem to probabilistic modular spaces by using a special condition. Shen and Chen [24] introduced the notion of modular space by using continuous t-norm and continuous t-conorm.

In 1998, Smarandache [21] characterized the new concept called neutrosophic logic and neutrosophic set and explored many results in it. In the idea of neutrosophic sets, there is T degree of membership, I degree of indeterminacy and F degree of non-membership Basset et al., explored the neutrosophic applications in dif and only different fields such as model for sustainable supply chain risk management, resource levelling problem in construction projects, decision making. In 2020, Kirisci et al [9] defined neutrosophic metric spaces as a generalization of Intuitionistic metric spaces and bring about fixed point theorems in complete neutrosophic metric spaces. In 2020, Sowndrarajan et al. [22] proved some fixed point results for contraction theorems in neutrosophic metric spaces.

The concept of neutrosophic modular space is first proposed in this paper. We investigate some topological properties and the existence of a relationship between an neutrosophic modular and an neutrosophic metric. The paper is organized as follows. First, we recall the fundamental definitions and the necessary examples of an neutrosophic metric space. In section 2, following the idea of modular spaces and the definition of an neutrosophic metric space, we give a new concept named neutrosophic modular space and give two examples to show that there does not exist a direct relationship between an neutrosophic modular and an neutrosophic metric. In section 3, a Hausdorff topology induced by a δ-homogeneous neutrosophic modular is defined, and several theorems on μ, ν, w completeness of the neutrosophic modular space are given. Finally, the well-known Baire's theorem and the uniform limit theorem are extended to neutrosophic modular spaces.

Definition 1.1. [**18**] Let Ξ be a non void set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm, if fuzzy sets $\mathfrak{P}, \mathfrak{I}, \mathfrak{K}$ on $\Xi^2 \times (0,\infty)$ such that for all $\acute{\mathfrak{e}}, \dot{\mathfrak{y}}, \grave{\mathfrak{z}} \in \Xi$ and $\breve{\varsigma}, \breve{\tau} > 0$ satisfies the following:
1. $\mathfrak{P}(\acute{\mathfrak{e}}, \dot{\mathfrak{y}}, \breve{\tau}) + \mathfrak{I}(\acute{\mathfrak{e}}, \dot{\mathfrak{y}}, \breve{\tau}) + \mathfrak{K}(\acute{\mathfrak{e}}, \grave{\mathfrak{k}}, \breve{\tau} \leqslant 3$,
2. $\mathfrak{P}(\acute{\mathfrak{e}}, \dot{\mathfrak{y}}, \breve{\tau}) > 0$,
3. $\mathfrak{P}(\acute{\mathfrak{e}}, \dot{\mathfrak{y}}, \breve{\tau}) = 1$ if and only if $\acute{\mathfrak{e}} = \dot{\mathfrak{y}}$,
4. $\mathfrak{P}(\acute{\mathfrak{e}}, \dot{\mathfrak{y}}, \breve{\tau}) = \mathfrak{P}(\dot{\mathfrak{y}}, \acute{\mathfrak{e}}, \breve{\tau})$,

5. $\mathfrak{P}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) * \mathfrak{P}(\grave{\mathfrak{y}}, \dot{\mathfrak{z}}, \breve{\varsigma}) \leqslant \mathfrak{P}(\acute{\mathfrak{e}}, \dot{\mathfrak{z}}, \breve{\tau} + \breve{\varsigma})$,
6. $\mathfrak{P}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, .) : (0, \infty) \to (0, 1]$ is continuous,
7. $\mathfrak{I}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) > 0$,
8. $\mathfrak{I}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = 0$ if and only if $\acute{\mathfrak{e}} = \grave{\mathfrak{y}}$,
9. $\mathfrak{I}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \mathfrak{I}(\grave{\mathfrak{y}}, \acute{\mathfrak{e}}, \breve{\tau})$,
10. $\mathfrak{I}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) \diamond \mathfrak{I}(\grave{\mathfrak{y}}, \dot{\mathfrak{z}}, \breve{\varsigma}) \geqslant \mathfrak{I}(\acute{\mathfrak{e}}, \dot{\mathfrak{z}}, \breve{\tau} + \breve{\varsigma})$,
11. $\mathfrak{I}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, .) : (0, \infty) \to (0, 1]$ is continuous.
12. $\mathfrak{K}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) > 0$,
13. $\mathfrak{K}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = 0$ if and only if $\acute{\mathfrak{e}} = \grave{\mathfrak{y}}$,
14. $\mathfrak{K}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \mathfrak{K}(\grave{\mathfrak{y}}, \acute{\mathfrak{e}}, \breve{\tau})$,
15. $\mathfrak{K}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) \diamond \mathfrak{K}(\grave{\mathfrak{y}}, \dot{\mathfrak{z}}, \breve{\varsigma}) \geqslant \mathfrak{K}(\acute{\mathfrak{e}}, \dot{\mathfrak{z}}, \breve{\tau} + \breve{\varsigma})$,
16. $\mathfrak{K}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, .) : (0, \infty) \to (0, 1]$ is continuous.

Then $(\mathfrak{P}, \mathfrak{I}, \mathfrak{K})$ is called a neutrosophic metric on Ξ.

Example 1.1. [**18**], Let $(\Xi, \mathfrak{d})$ be a metric space.
Denote $\mathfrak{a} * \mathfrak{b} = \mathfrak{a}\mathfrak{b}$ and $\mathfrak{a} \diamond \mathfrak{b} = \min\{1, \mathfrak{a} + \mathfrak{b}\}$ for all $\mathfrak{a}, \mathfrak{b} \in [0, 1]$ and let $\mathfrak{P}_{\mathfrak{d}}$, $\mathfrak{I}_{\mathfrak{d}}$ and $\mathfrak{K}_{\mathfrak{d}}$ be fuzzy sets on $\Xi^2 \times (0, \infty)$ defined as follows:
$\mathfrak{P}_{\mathfrak{d}}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \frac{\mathfrak{h}\breve{\tau}^n}{\mathfrak{h}\breve{\tau}^n + \mathfrak{m}\mathfrak{d}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}})}, \mathfrak{I}_{\mathfrak{d}}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \frac{\mathfrak{d}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}})}{\mathfrak{k}\breve{\tau}^n + \mathfrak{m}\mathfrak{d}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}})}, \mathfrak{K}_{\mathfrak{d}}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \frac{\mathfrak{d}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}})}{l\breve{\tau}^n + \mathfrak{m}\mathfrak{d}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}})}$
for all $\mathfrak{h}, \mathfrak{k}, l, \mathfrak{m}, \mathfrak{n} \in \mathbb{R}^+$. Then $(\Xi, \mathfrak{P}_{\mathfrak{d}}, \mathfrak{I}_{\mathfrak{d}}, \mathfrak{K}_{\mathfrak{d}} *, \diamond, \odot)$ is a neutrosophic metric space.

2. Neutrosophic modular spaces

In this section, we introduce the concept of a neutrosophic modular space by using continuous t-norm and continuous t-conorm. We investigate the relationship between a neutrosophic modular and a neutrosophic metric.

Definition 2.1. A 7-tuple $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ is said to be an neutrosophic modular space if Ξ is a real or complex vector space, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and $\dot{\lambda}, \ddot{\pi}, \dddot{\varpi}$ are sets on $\Xi \times (0, \infty)$ such that for all $\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \dot{\mathfrak{z}} \in \Xi, \breve{\varsigma}, \breve{\tau} > 0$ and $\hat{\epsilon}, \breve{\kappa} \geqslant 0$ with $\hat{\epsilon} + \breve{\kappa} = 1$ followings hold:
1. $\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\tau}) + \ddot{\pi}(\acute{\mathfrak{e}}, \breve{\tau}) + \dddot{\varpi}(\acute{\mathfrak{e}}, \breve{\tau}) \leqslant 3$,
2. $\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\tau}) > 0$,
3. $\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\tau}) = 1$ if and only if $\acute{\mathfrak{e}} = 0$,
4. $\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\tau}) = \dot{\lambda}(-\acute{\mathfrak{e}}, \breve{\tau})$,
5. $\dot{\lambda}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}}, \breve{\varsigma} + \breve{\tau}) \geqslant \dot{\lambda}(\acute{\mathfrak{e}}, \breve{\varsigma}) * \dot{\lambda}(\grave{\mathfrak{y}}, \breve{\tau})$,
6. $\dot{\lambda}(\acute{\mathfrak{e}}, .) : (0, \infty) \to (0, 1]$ is continuous,
7. $\ddot{\pi}(\acute{\mathfrak{e}}, \breve{\tau}) > 0$,
8. $\ddot{\pi}(\acute{\mathfrak{e}}, \breve{\tau}) = 0$ if and only if $\acute{\mathfrak{e}} = 0$,
9. $\ddot{\pi}(\acute{\mathfrak{e}}, \breve{\tau}) = \ddot{\pi}(-\acute{\mathfrak{e}}, \breve{\tau})$,
10. $\ddot{\pi}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}}, \breve{\varsigma} + \breve{\tau}) \leqslant \ddot{\pi}(\acute{\mathfrak{e}}, \breve{\varsigma}) \diamond \ddot{\pi}(\grave{\mathfrak{y}}, \breve{\tau})$,
11. $\ddot{\pi}(\acute{\mathfrak{e}}, .) : (0, \infty) \to (0, 1]$ is continuous.
12. $\dddot{\varpi}(\acute{\mathfrak{e}}, \breve{\tau}) > 0$,
13. $\dddot{\varpi}(\acute{\mathfrak{e}}, \breve{\tau}) = 0$ if and only if $\acute{\mathfrak{e}} = 0$,
14. $\dddot{\varpi}(\acute{\mathfrak{e}}, \breve{\tau}) = \dddot{\varpi}(-\acute{\mathfrak{e}}, \breve{\tau})$,
15. $\dddot{\varpi}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}}, \breve{\varsigma} + \breve{\tau}) \leqslant \dddot{\varpi}(\acute{\mathfrak{e}}, \breve{\varsigma}) \diamond \dddot{\varpi}(\grave{\mathfrak{y}}, \breve{\tau})$,

16. $\dddot{\varpi}(\mathfrak{e}, .) : (0, \infty) \to (0, 1]$ is continuous.
Then $(\dot{\lambda}, \ddot{\pi}, \dddot{\varpi})$ is called a neutrosophic modular or neutrosophic $\hat{\mathfrak{F}}$-modular on Ξ. The 7-tuple $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ is called δ-homogeneous, where $\delta \in (0, 1]$, if for each $\acute{\mathfrak{e}} \in \Xi, \breve{\tau} > 0$ and $\tilde{\Upsilon} \in \mathbb{R} - \{0\}$,
$\dot{\lambda}(\tilde{\Upsilon}\acute{\mathfrak{e}}, \breve{\tau}) = \dot{\lambda}\left(\acute{\mathfrak{e}}, \frac{t}{|\tilde{\Upsilon}|^{\delta}}\right), \ddot{\pi}(\tilde{\Upsilon}\acute{\mathfrak{e}}, \breve{\tau}) = \ddot{\pi}\left(\acute{\mathfrak{e}}, \frac{\breve{\tau}}{|\tilde{\Upsilon}|^{\delta}}\right), \dddot{\varpi}(\tilde{\Upsilon}\acute{\mathfrak{e}}, \breve{\tau}) = \dddot{\varpi}\left(\acute{\mathfrak{e}}, \frac{\breve{\tau}}{|\tilde{\Upsilon}|^{\delta}}\right).$

Remark 2.1. (i) If $(\Xi, \dot{\lambda}, *)$ is a $\hat{\mathfrak{F}}$-modular space, then $(\Xi, \dot{\lambda}, 1-\dot{\lambda}, 1-\dot{\lambda}, *, \diamond, \odot)$ is an N$\hat{\mathfrak{F}}$M such that for any $\mathfrak{a}, \mathfrak{b} \in [0, 1], \mathfrak{a}\diamond\mathfrak{b} = 1 - ((1 - \mathfrak{a}) * (1 - \mathfrak{b}))$.
(ii) In N$\hat{\mathfrak{F}}$M $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$, for all $\acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \Xi, \dot{\lambda}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, .)$ is non-decreasing and $\ddot{\pi}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, .)$, $\dddot{\varpi}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, .)$ are non-increasing.

Example 2.1. Let $(\Xi, \dot{\varrho})$ be a modular space.
Consider $\mathfrak{a} * \mathfrak{b} = \mathfrak{a}\mathfrak{b}$, $\mathfrak{a}\diamond\mathfrak{b} = \min\{1, \mathfrak{a} + \mathfrak{b}\}$ as well as $\mathfrak{a} \odot \mathfrak{b} = \min\{1, \mathfrak{a} + \mathfrak{b}\}$ for all $\mathfrak{a}, \mathfrak{b} \in [0, 1]$, in addition to define sets $\dot{\lambda}_{\dot{\varrho}}$, $\ddot{\pi}_{\dot{\varrho}}$ and $\dddot{\varpi}_{\dot{\varrho}}$on $\Xi \times (0, \infty)$ according to,
$\dot{\lambda}_{\dot{\varrho}}(\acute{\mathfrak{e}}, \breve{\tau}) = \frac{\mathfrak{h}\breve{\tau}^n}{\mathfrak{h}\breve{\tau}^n + \mathfrak{m}\dot{\varrho}(\acute{\mathfrak{e}})}, \ddot{\pi}_{\dot{\varrho}}(\acute{\mathfrak{e}}, \breve{\tau}) = \frac{\dot{\varrho}(\acute{\mathfrak{e}})}{\mathfrak{k}\breve{\tau}^n + \mathfrak{m}\dot{\varrho}(\acute{\mathfrak{e}})}, \dddot{\varpi}_{\dot{\varrho}}(\acute{\mathfrak{e}}, \breve{\tau}) = \frac{\mathfrak{m}\dot{\varrho}(\acute{\mathfrak{e}})}{\mathfrak{k}\breve{\tau}^n}$
for all $\mathfrak{h}, \mathfrak{k} \in \mathbb{R}^+$ and $\mathfrak{m}, \mathfrak{n} \in \mathbb{N}$. Then $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ is a neutrosophic $\hat{\mathfrak{F}}$-modular space (N$\hat{\mathfrak{F}}$M). We look into condition (5) within Definition (2.1). In this case, let $\hat{\epsilon}, \breve{\kappa} \geqslant 0$ with $\hat{\epsilon} + \breve{\kappa} = 1$, because $\dot{\varrho}$ is modular, we get

$$\dot{\varrho}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}}) \leqslant \dot{\varrho}(\acute{\mathfrak{e}}) + \dot{\varrho}(\grave{\mathfrak{y}}), \tag{2.1}$$

for every $\acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \Xi$. As a result

$$\begin{aligned}
\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\varsigma}) * \dot{\lambda}(\grave{\mathfrak{y}}, \breve{\tau}) &= \frac{\mathfrak{h}\breve{\varsigma}^n}{\mathfrak{h}\breve{\varsigma}^n + \mathfrak{m}\dot{\varrho}(\acute{\mathfrak{e}})} * \frac{\mathfrak{h}\breve{\tau}^n}{\mathfrak{h}\breve{\tau}^n + \mathfrak{m}\dot{\varrho}(\grave{\mathfrak{y}})} \\
&= \frac{\mathfrak{h}^2\breve{\varsigma}^n\breve{\tau}^n}{(\mathfrak{h}\breve{\varsigma}^n + \mathfrak{m}\dot{\varrho}(\acute{\mathfrak{e}}))(\mathfrak{h}\breve{\tau}^n + \mathfrak{m}\dot{\varrho}(\grave{\mathfrak{y}}))} \\
&\leqslant \frac{\mathfrak{h}\breve{\varsigma}^n\breve{\tau}^n}{\mathfrak{h}\breve{\varsigma}^n\breve{\tau}^n + \mathfrak{m}(\breve{\tau}^n\dot{\varrho}(\acute{\mathfrak{e}}) + \breve{\varsigma}^n\dot{\varrho}(\grave{\mathfrak{y}}))}.
\end{aligned}$$

Without lossing of generality, we assume that $\breve{\tau} \leqslant \breve{\varsigma}$. Using (2.1) we then obtain

$$\begin{aligned}
\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\varsigma}) * \dot{\lambda}(y, \breve{\tau}) &\leqslant \frac{\mathfrak{h}\breve{\varsigma}^n}{\mathfrak{h}\breve{\varsigma}^n + \mathfrak{m}\dot{\varrho}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}})} \\
&\leqslant \frac{\mathfrak{h}(\breve{\varsigma} + \breve{\tau})^n}{\mathfrak{h}(\breve{\varsigma} + \breve{\tau})^n + \mathfrak{m}\dot{\varrho}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}})} \\
&= \dot{\lambda}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}}, \breve{\varsigma} + \breve{\tau}).
\end{aligned}$$

Remark 2.2. By getting $\mathfrak{h} = \mathfrak{k} = \mathfrak{m} = \mathfrak{n} = 1$, from Example (2.1), we obtain $\dot{\lambda}_{\dot{\varrho}}(\acute{\mathfrak{e}}, \breve{\tau}) = \frac{\breve{\tau}}{\breve{\tau} + \dot{\varrho}(\acute{\mathfrak{e}})}, \ddot{\pi}_{\dot{\varrho}}(\acute{\mathfrak{e}}, \breve{\tau}) = \frac{\dot{\varrho}(\acute{\mathfrak{e}})}{\breve{\tau} + \dot{\varrho}(\acute{\mathfrak{e}})}, \dddot{\varpi}_{\dot{\varrho}}(\acute{\mathfrak{e}}, \breve{\tau}) = \frac{\dot{\varrho}(\acute{\mathfrak{e}})}{\breve{\tau}}$. This N$\hat{\mathfrak{F}}$M is called the standard N$\hat{\mathfrak{F}}$M.

It should be noted that, in general, a neutrosophic modular and a neutrosophic metric do not necessarily induce mutually a metric when the triangular norm is the same one. In essence, the neutrosophic modular and neutrosophic metric can be viewed as two different characterizations for the same set. Next, we give two examples to show that there does not exist a direct relationship between neutrosophic modular and an neutrosophic metric. In fact, the neutrosophic modular and the neutrosophic metric can be viewed as two different characterizations for the same set.

EXAMPLE 2.2. Let $\Xi = \mathbb{R}$ in addition to apply $\dot{\varrho}(\acute{\mathfrak{e}}) = |\acute{\mathfrak{e}}|$, afterwards $\dot{\varrho}$ is modular on Ξ. Apply $\mathfrak{a} * \mathfrak{b} = \min\{\mathfrak{a}, \mathfrak{b}\}$, $\mathfrak{a} \diamond \mathfrak{b} = 1 - ((1-\mathfrak{a}) * (1-\mathfrak{b}))$ and
$\mathfrak{a} \odot \mathfrak{b} = 1 - ((1-\mathfrak{a}) * (1-\mathfrak{b}))$ or $\mathfrak{a} \diamond \mathfrak{b} = \max\{\mathfrak{a}, \mathfrak{b}\}, \mathfrak{a} \odot \mathfrak{b} = \max\{\mathfrak{a}, \mathfrak{b}\}$. For every $\breve{\tau} \in (0, \infty)$ and $\acute{\mathfrak{e}} \in \Xi$, define $\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\tau}) = \frac{\breve{\tau}}{\breve{\tau} + |\acute{\mathfrak{e}}|}$. Then $(\Xi, \dot{\lambda}, *)$ is an $\hat{\mathfrak{F}}$-modular space and so by Remark (2.1),
$(\Xi, \dot{\lambda}, 1 - \dot{\lambda}, , 1 - \dot{\lambda}, *, \diamond, \odot)$ is an neutrosophic $\hat{\mathfrak{F}}$-modular space. However, if we set $\mathfrak{P}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \dot{\lambda}(\acute{\mathfrak{e}} - \grave{\mathfrak{y}}, \breve{\tau}) = \frac{\breve{\tau}}{\breve{\tau} + |\acute{\mathfrak{e}} - \grave{\mathfrak{y}}|}, \mathfrak{I}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \frac{|\acute{\mathfrak{e}} - \grave{\mathfrak{y}}|}{\breve{\tau} + |\acute{\mathfrak{e}} - \grave{\mathfrak{y}}|}$, and $\mathfrak{K}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \frac{|\acute{\mathfrak{e}} - \grave{\mathfrak{y}}|}{\breve{\tau}}$,
when t-norm as well as the t-conorm defined as
$\mathfrak{a} * \mathfrak{b} = \min\{\mathfrak{a}, \mathfrak{b}\}$, $\mathfrak{a} \diamond \mathfrak{b} = \max\{\mathfrak{a}, \mathfrak{b}\}$ and $\mathfrak{a} \odot \mathfrak{b} = \max\{\mathfrak{a}, \mathfrak{b}\}$ respectively.
Remark 2.1 suggests that $(\mathfrak{P}, \mathfrak{I}, \mathfrak{K})$ is not a neutrosophic metric.

EXAMPLE 2.3. Let $\Xi = \mathbb{R}$. Take t-norm $\mathfrak{a} * \mathfrak{b} = \min\{\mathfrak{a}, \mathfrak{b}\}$, t-conorm
$\mathfrak{a} \diamond \mathfrak{b} = \mathfrak{a} + \mathfrak{b} - \mathfrak{a}\mathfrak{b}$ and $\mathfrak{a} \odot \mathfrak{b} = \mathfrak{a} + \mathfrak{b} - \mathfrak{a}\mathfrak{b}$. For every $\acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \Xi$ and $\breve{\tau} \in (0, \infty)$, we specify

$$\mathfrak{P}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \begin{cases} 1, & \acute{\mathfrak{e}} = \grave{\mathfrak{y}} \\ \frac{1}{2}, & \acute{\mathfrak{e}} \neq \grave{\mathfrak{y}}, \acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \mathbb{Z} \\ \frac{1}{4}, & \acute{\mathfrak{e}} \in \mathbb{Z}, \grave{\mathfrak{y}} \in \mathbb{R} \backslash \mathbb{Z} \, \text{or} \, \acute{\mathfrak{e}} \in \mathbb{R} \backslash \mathbb{Z}, \grave{\mathfrak{y}} \in \mathbb{Z} \\ \frac{1}{4}, & \acute{\mathfrak{e}} \neq \grave{\mathfrak{y}}, \acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \mathbb{R} \backslash \mathbb{Z}, \end{cases}$$

$$\mathfrak{I}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \begin{cases} 0, & \acute{\mathfrak{e}} = \grave{\mathfrak{y}} \\ \frac{1}{4}, & \acute{\mathfrak{e}} \neq \grave{\mathfrak{y}}, \acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \mathbb{Z} \\ \frac{1}{2}, & \acute{\mathfrak{e}} \in \mathbb{Z}, \grave{\mathfrak{y}} \in \mathbb{R} \backslash \mathbb{Z} \, \text{or} \, \acute{\mathfrak{e}} \in \mathbb{R} \backslash \mathbb{Z}, \grave{\mathfrak{y}} \in \mathbb{Z} \\ \frac{1}{2}, & \acute{\mathfrak{e}} \neq \grave{\mathfrak{y}}, \acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \mathbb{R} \backslash \mathbb{Z}, \end{cases} \quad \text{and}$$

$$\mathfrak{K}(\acute{\mathfrak{e}}, \grave{\mathfrak{y}}, \breve{\tau}) = \begin{cases} 0, & \acute{\mathfrak{e}} = \grave{\mathfrak{y}} \\ \frac{1}{4}, & \acute{\mathfrak{e}} \neq \grave{\mathfrak{y}}, \acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \mathbb{Z} \\ \frac{1}{2}, & \acute{\mathfrak{e}} \in \mathbb{Z}, \grave{\mathfrak{y}} \in \mathbb{R} \backslash \mathbb{Z} \, \text{or} \, \acute{\mathfrak{e}} \in \mathbb{R} \backslash \mathbb{Z}, \grave{\mathfrak{y}} \in \mathbb{Z} \\ \frac{1}{2}, & \acute{\mathfrak{e}} \neq \grave{\mathfrak{y}}, \acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \mathbb{R} \backslash \mathbb{Z}, \end{cases}$$

The fact that it exists $(\mathfrak{P}, \mathfrak{I}, \mathfrak{K}, *, \diamond, \odot)$ is a neutrosophic metric on Ξ can easily be demonstrated. Now, set

$$\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\tau}) = \begin{cases} 1, & \acute{\mathfrak{e}} = 0 \\ \frac{1}{2}, & \acute{\mathfrak{e}} \in \mathbb{Z} \backslash \{0\} \\ \frac{1}{4}, & \acute{\mathfrak{e}} \in \mathbb{R} \backslash \mathbb{Z} \end{cases}, \ddot{\pi}(\acute{\mathfrak{e}}, \breve{\tau}) = \begin{cases} 0, & \acute{\mathfrak{e}} = 0 \\ \frac{1}{4}, & \acute{\mathfrak{e}} \in \mathbb{Z} \backslash \{0\} \\ \frac{1}{2}, & \acute{\mathfrak{e}} \in \mathbb{R} \backslash \mathbb{Z}. \end{cases} \quad \text{and}$$

$$\dddot{\varpi}(\acute{\mathfrak{e}}, \breve{\tau}) = \begin{cases} 0, & \acute{\mathfrak{e}} = 0 \\ \frac{1}{4}, & \acute{\mathfrak{e}} \in \mathbb{Z} \backslash \{0\} \\ \frac{1}{2}, & \acute{\mathfrak{e}} \in \mathbb{R} \backslash \mathbb{Z} \end{cases}$$

When we choose $\hat{\epsilon} = \frac{\sqrt{2}}{2}, \breve{\kappa} = 1 - \hat{\epsilon}, \acute{\mathfrak{e}} \neq \grave{\mathfrak{y}}$, and $\acute{\mathfrak{e}}, \grave{\mathfrak{y}} \in \mathbb{Z}$, then $\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}} \in \mathbb{R} \backslash \mathbb{Z}$.
Hence for each $\breve{\varsigma}, \breve{\tau} > 0$, we have $\dot{\lambda}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}}, \breve{\varsigma} + \breve{\tau}) = \frac{1}{4}$, but $\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\varsigma}) * \dot{\lambda}(\grave{\mathfrak{y}}, \breve{\tau}) = \frac{1}{2}$.
Also $\ddot{\pi}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}}, \breve{\varsigma} + \breve{\tau}) = \frac{1}{2}$, $\dddot{\varpi}(\hat{\epsilon}\acute{\mathfrak{e}} + \breve{\kappa}\grave{\mathfrak{y}}, \breve{\varsigma} + \breve{\tau}) = \frac{1}{2}$, but $\ddot{\pi}(\acute{\mathfrak{e}}, \breve{\varsigma}) \diamond \ddot{\pi}(\grave{\mathfrak{y}}, \breve{\tau}) = \frac{1}{4}$.
Therefore $(\dot{\lambda}, \ddot{\pi}, \dddot{\varpi})$ is not an NM on Ξ.

3. Topology induced by δ-homogeneous neutrosophic modular spaces

In this section, we define a topology induced by a δ-homogeneous N $\hat{\mathfrak{F}}$M and investigate some topological properties in δ-homogeneous N$\hat{\mathfrak{F}}$M. The results obtained in this section are an extension of the results presented in [**24**] to N$\hat{\mathfrak{F}}$M.

Definition 3.1. Let $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \Diamond, \odot)$ be an N$\hat{\mathfrak{F}}$M and let $\acute{\mathfrak{e}} \in \Xi, \mathfrak{r} \in (0,1)$ and $\breve{\tau} > 0$. Then the $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-ball with center $\acute{\mathfrak{e}}$ and radius $\mathfrak{r}$ with respect to $\breve{\tau}$ is defined as $\mathfrak{B}(\acute{\mathfrak{e}}, \mathfrak{r}, \breve{\tau}) = \{\dot{\mathfrak{y}} \in \Xi : \dot{\lambda}(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \breve{\tau}) > 1 - \mathfrak{r}, \ddot{\pi}(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \breve{\tau}) < \mathfrak{r}, \dddot{\varpi}(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \breve{\tau}) < \mathfrak{r}\}$. Let $\mathfrak{E} \subseteq \Xi$. An element $\acute{\mathfrak{e}} \in \mathfrak{E}$ is called a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-interior point of $\mathfrak{E}$ if there exist $\mathfrak{r} \in (0,1)$ and $\breve{\tau} > 0$ such that $\mathfrak{B}(\acute{\mathfrak{e}}, \mathfrak{r}, \breve{\tau}) \subseteq \mathfrak{E}$. We say that $\mathfrak{E}$ is a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-open set in Ξ if and only if every element of $\mathfrak{E}$ is a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-interior point. Note that each open set in an N$\hat{\mathfrak{F}}$Mis not a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-ball in general.

Example 3.1. Let $\Xi = \mathbb{R}$ and let $\dot{\varrho}, \dot{\lambda}, *$ and $\Diamond$ be as in Example (2.2. Consider $\mathfrak{V} = \{\acute{\mathfrak{e}} \in \mathbb{R} : 0 < \acute{\mathfrak{e}} < 1\} \cup \{\acute{\mathfrak{e}} \in \mathbb{R} : 1 < \acute{\mathfrak{e}} < 2\}$. Then $\mathfrak{V}$ is an open set in $(\mathbb{R}, \dot{\lambda}, 1 - \dot{\lambda}, *, \Diamond)$, but it is not a $\dot{\lambda} - (1 - \dot{\lambda})$-ball. In fact, the $\dot{\lambda} - (1 - \dot{\lambda})$-ball in $(\mathbb{R}, \dot{\lambda}, 1 - \dot{\lambda}, *, \Diamond)$ with center $\acute{\mathfrak{e}}$ and radius $\mathfrak{r}$ is as follows.

$$\begin{aligned}\mathfrak{B}(\acute{\mathfrak{e}}, \mathfrak{r}, \breve{\tau}) &= \left\{\dot{\mathfrak{y}} \in \mathbb{R} : \frac{\breve{\tau}}{\breve{\tau} + |\acute{\mathfrak{e}} - \dot{\mathfrak{y}}|} > 1 - \mathfrak{r}, \frac{|\acute{\mathfrak{e}} - \dot{\mathfrak{y}}|}{\breve{\tau} + |\acute{\mathfrak{e}} - \dot{\mathfrak{y}}|} < \mathfrak{r}\right\} \\ &= \left\{\dot{\mathfrak{y}} \in \mathbb{R} : |\acute{\mathfrak{e}} - \dot{\mathfrak{y}}| < \frac{\mathfrak{r}}{1 - \mathfrak{r}}\breve{\tau}\right\}.\end{aligned}$$

Theorem 3.1. *Each $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-ball in a δ-homogeneous N$\hat{\mathfrak{F}}$Mis an open set.*

Proof. Let $\mathfrak{B}(\acute{\mathfrak{e}}, \mathfrak{r}, \breve{\tau})$ be a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-ball and $\dot{\mathfrak{y}} \in \mathfrak{B}(\acute{\mathfrak{e}}, \mathfrak{r}, \breve{\tau})$. Then

$$\dot{\lambda}(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \breve{\tau}) > 1 - \mathfrak{r}, \ddot{\pi}(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \breve{\tau}) < \mathfrak{r} \text{ and } \dddot{\varpi}(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \breve{\tau}) < \mathfrak{r}.$$

Put $\breve{\tau} = 2\breve{\tau}_1$. Since $\dot{\lambda}(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, .)$ and $\ddot{\pi}(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, .)$ are continuous, there exists $\xi_{\dot{\mathfrak{y}}} > 0$ such that

$$\dot{\lambda}\left(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \frac{\breve{\tau}_1 - \xi_{\dot{\mathfrak{y}}}}{2^{\delta-1}}\right) > 1 - \mathfrak{r}, \ddot{\pi}\left(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \frac{\breve{\tau}_1 - \xi_{\dot{\mathfrak{y}}}}{2^{\delta-1}}\right) < \mathfrak{r} \text{ and } \dddot{\varpi}\left(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \frac{\breve{\tau}_1 - \xi_{\dot{\mathfrak{y}}}}{2^{\delta-1}}\right) < \mathfrak{r}.$$

For some $\varepsilon > 0$ with $\frac{\breve{\tau}_1 - \varepsilon}{2^{\delta-1}} > 0$ and $\frac{\varepsilon}{2^{\delta-1}} \in (0, \xi_{\dot{\mathfrak{y}}})$, put $\mathfrak{r}_0 = \dot{\lambda}\left(\acute{\mathfrak{e}} - \dot{\mathfrak{y}}, \frac{\breve{\tau}_1 - \varepsilon}{2^{\delta-1}}\right)$. Since $\mathfrak{r}_0 > \mathfrak{r} - 1$, there exists $\breve{\varsigma} \in (0,1)$ such that $\mathfrak{r}_0 > 1 - \breve{\varsigma} > 1 - \mathfrak{r}$, by Lemma, we can choose $\mathfrak{r}_1 \in (0,1)$ such that $\mathfrak{r}_0 * \mathfrak{r}_0 \geqslant 1 - \breve{\varsigma}, (1 - \mathfrak{r}_0) \Diamond (1 - \mathfrak{r}_0) \leqslant \breve{\varsigma}$.
Put $\mathfrak{r}_3 = \max\{\mathfrak{r}_1, \mathfrak{r}_2\}$. We show that $\mathfrak{B}\left(\dot{\mathfrak{y}}, 1 - \mathfrak{r}_3, \frac{\varepsilon}{2^{\delta-1}}\right) \subseteq \mathfrak{B}(\acute{\mathfrak{e}}, \mathfrak{r}, 2\breve{\tau}_1)$.
Suppose that $\dot{\mathfrak{z}} \in \mathfrak{B}\left(\dot{\mathfrak{y}}, 1 - \mathfrak{r}_3, \frac{\varepsilon}{2^{\delta-1}}\right)$ then

$$\dot{\lambda}\left(\dot{\mathfrak{y}} - \dot{\mathfrak{z}}, \frac{\varepsilon}{2^{\delta-1}}\right) > \mathfrak{r}_3, \ddot{\pi}\left(\dot{\mathfrak{y}} - \dot{\mathfrak{z}}, \frac{\varepsilon}{2^{\delta-1}}\right) < 1 - \mathfrak{r}_3 \text{ and } \dddot{\varpi}\left(\dot{\mathfrak{y}} - \dot{\mathfrak{z}}, \frac{\varepsilon}{2^{\delta-1}}\right) < 1 - \mathfrak{r}_3.$$

Therefore

$$\begin{aligned}
\dot{\lambda}(\mathfrak{\acute{e}}-\mathfrak{z},\breve{\tau}) &= \dot{\lambda}(\mathfrak{\acute{e}}-\mathfrak{z},2\breve{\tau}_1) \geqslant \dot{\lambda}(2(\mathfrak{\acute{e}}-\mathfrak{\dot{y}}),2(\breve{\tau}_1-\varepsilon)) * \dot{\lambda}(2(\mathfrak{\dot{y}}-\mathfrak{z}),2\varepsilon)\\
&= \dot{\lambda}\left(\mathfrak{\acute{e}}-\mathfrak{\dot{y}},\frac{\breve{\tau}_1-\varepsilon}{2^{\delta-1}}\right) * \dot{\lambda}\left(\mathfrak{\dot{y}}-\mathfrak{z},\frac{\varepsilon}{2^{\delta-1}}\right)\\
&\geqslant \mathfrak{r}_0 * \mathfrak{r}_1 \geqslant 1-\breve{\varsigma} > 1-\mathfrak{r},\\
\ddot{\pi}(\mathfrak{\acute{e}}-\mathfrak{z},\breve{\tau}) &= \ddot{\pi}(\mathfrak{\acute{e}}-\mathfrak{z},2\breve{\tau}_1) \leqslant \ddot{\pi}(2(\mathfrak{\acute{e}}-\mathfrak{\dot{y}}),2(\breve{\tau}_1-\varepsilon))\diamond\ddot{\pi}(2(\mathfrak{\dot{y}}-\mathfrak{z}),2\varepsilon)\\
&= \ddot{\pi}\left(\mathfrak{\acute{e}}-\mathfrak{\dot{y}},\frac{\breve{\tau}_1-\varepsilon}{2^{\delta-1}}\right)\diamond\ddot{\pi}\left(\mathfrak{\dot{y}}-\mathfrak{z},\frac{\varepsilon}{2^{\delta-1}}\right)\\
&< (1-\mathfrak{r}_0)\diamond(1-\mathfrak{r}_3) \leqslant (1-\mathfrak{r}_0)\diamond(1-\mathfrak{r}_2) \leqslant \breve{\varsigma} < \mathfrak{r},\\
\ddot{\pi}(\mathfrak{\acute{e}}-\mathfrak{z},\breve{\tau}) &= \dddot{\varpi}(\mathfrak{\acute{e}}-\mathfrak{z},2\breve{\tau}_1) \leqslant \dddot{\varpi}(2(\mathfrak{\acute{e}}-\mathfrak{\dot{y}}),2(\breve{\tau}_1-\varepsilon))\diamond\dddot{\varpi}(2(\mathfrak{\dot{y}}-\mathfrak{z}),2\varepsilon)\\
&= \dddot{\varpi}\left(\mathfrak{\acute{e}}-\mathfrak{\dot{y}},\frac{\breve{\tau}_1-\varepsilon}{2^{\delta-1}}\right)\diamond\dddot{\varpi}\left(\mathfrak{\dot{y}}-\mathfrak{z},\frac{\varepsilon}{2^{\delta-1}}\right)\\
&< (1-\mathfrak{r}_0)\diamond(1-\mathfrak{r}_3) \leqslant (1-\mathfrak{r}_0)\diamond(1-\mathfrak{r}_2) \leqslant \breve{\varsigma} < \mathfrak{r}.
\end{aligned}$$

Therefore $\mathfrak{z} \in \mathfrak{B}(\mathfrak{\acute{e}},\mathfrak{r},\breve{\tau})$ and hence $\mathfrak{B}\left(\mathfrak{\dot{y}},1-\mathfrak{r}_3,\frac{\varepsilon}{2^{\delta-1}}\right) \subseteq \mathfrak{B}(\mathfrak{\acute{e}},\mathfrak{r},\breve{\tau})$.
Now, we define a topology on a δ-homogeneous N$\hat{\mathfrak{F}}$M. □

DEFINITION 3.2. Let $(\Xi,\dot{\lambda},\ddot{\pi},\dddot{\varpi},*,\diamond,\odot)$ be a δ-homogeneous N$\hat{\mathfrak{F}}$M. Define $\tau(\dot{\lambda},\ddot{\pi},\dddot{\varpi}) = \{\mathfrak{V} \subseteq \Xi : \forall\mathfrak{\acute{e}} \in \mathfrak{V}, \exists\breve{\tau} > 0, \mathfrak{r} \in (0,1); \mathfrak{B}(\mathfrak{\acute{e}},\mathfrak{r},\breve{\tau}) \subseteq \mathfrak{V}\}$. Then $\tau(\dot{\lambda},\ddot{\pi},\dddot{\varpi})$ is a topology on Ξ.

REMARK 3.1. Since the family of $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-balls $\left\{\mathfrak{B}\left(\mathfrak{\acute{e}},\frac{1}{n},\frac{1}{n}\right) : n \in \mathbb{N}\right\}$ is a local base at $\mathfrak{\acute{e}}$, the topology $\tau(\dot{\lambda},\ddot{\pi},\dddot{\varpi})$ is first countable.

EXAMPLE 3.2. Let $\Xi = \mathbb{R}$ and let $\dot{\varrho},\dot{\lambda},*$ and $\diamond$ be as in Example (2.2). Then the set of all $\{(\mathfrak{a},\mathfrak{b}) : \mathfrak{a},\mathfrak{b} \in \mathbb{R}\}$ induces a topology on $(\mathbb{R},\dot{\lambda},1-\dot{\lambda},*,\diamond)$.

THEOREM 3.2. *Every δ-homogeneous N$\hat{\mathfrak{F}}$M is Hausdorff.*

PROOF. Let $\mathfrak{\acute{e}},\mathfrak{\dot{y}}$ be two distinct points in δ-homogeneous N$\hat{\mathfrak{F}}$M $(\Xi,\dot{\lambda},\ddot{\pi},\dddot{\varpi},*,\diamond,\odot)$. Then for all $\breve{\tau} > 0, 0 < \dot{\lambda}(\mathfrak{\acute{e}}-\mathfrak{\dot{y}},\breve{\tau}) < 1, 0 < \ddot{\pi}(\mathfrak{\acute{e}}-\mathfrak{\dot{y}},\breve{\tau}) < 1$.
Put $\mathfrak{r}_1 = \dot{\lambda}(\mathfrak{\acute{e}}-\mathfrak{\dot{y}},\breve{\tau}), \mathfrak{r}_2 = \ddot{\pi}(\mathfrak{\acute{e}}-\mathfrak{\dot{y}},\breve{\tau})$ and $\mathfrak{r} = \max\{\mathfrak{r}_1,\mathfrak{r}_2\}$.
For $\mathfrak{r}_0 \in (\mathfrak{r},1)$, there are $\mathfrak{r}_3,\mathfrak{r}_4$such that $\mathfrak{r}_3 * \mathfrak{r}_3 \geqslant \mathfrak{r}_0, (1-\mathfrak{r}_4)\diamond(1-\mathfrak{r}_4) \leqslant 1-\mathfrak{r}_0$.
Put $\mathfrak{r}_5 = \max\{\mathfrak{r}_3,\mathfrak{r}_4\}$. Then $\mathfrak{B}\left(\mathfrak{\acute{e}},1-\mathfrak{r}_5,\frac{\breve{\tau}}{2^{\delta+1}}\right)\bigcap\mathfrak{B}\left(\mathfrak{\dot{y}},1-\mathfrak{r}_5,\frac{\breve{\tau}}{2^{\aleph+1}}\right) = \emptyset$.

Otherwise, if there exists $\mathfrak{z} \in \mathfrak{B}\left(\acute{\mathfrak{e}}, 1-\mathfrak{r}_5, \frac{\breve{\tau}}{2^{\delta+1}}\right) \bigcap \mathfrak{B}\left(\grave{\mathfrak{y}}, 1-\mathfrak{r}_5, \frac{\breve{\tau}}{2^{\delta+1}}\right)$, then

$$\begin{aligned}
\mathfrak{r}_1 = \dot{\lambda}(\acute{\mathfrak{e}}-\grave{\mathfrak{y}}, \breve{\tau}) &\geqslant \dot{\lambda}\left(2(\acute{\mathfrak{e}}-\mathfrak{z}), \frac{\breve{\tau}}{2}\right) * \dot{\lambda}\left(2(\mathfrak{z}-\grave{\mathfrak{y}}), \frac{\breve{\tau}}{2}\right) \\
&= \dot{\lambda}\left(\acute{\mathfrak{e}}-\mathfrak{z}, \frac{\breve{\tau}}{2^{\delta+1}}\right) * \dot{\lambda}\left(\mathfrak{z}-\grave{\mathfrak{y}}, \frac{\breve{\tau}}{2^{\delta+1}}\right) \\
&\geqslant \mathfrak{r}_5 * \mathfrak{r}_5 \geqslant \mathfrak{r}_3 * \mathfrak{r}_3 \geqslant \mathfrak{r}_0 > \mathfrak{r}_1, \text{ and} \\
\mathfrak{r}_2 = \ddot{\pi}(\acute{\mathfrak{e}}-\grave{\mathfrak{y}}, \breve{\tau}) &\leqslant \ddot{\pi}\left(2(\acute{\mathfrak{e}}-\mathfrak{z}), \frac{\breve{\tau}}{2}\right) \diamond \ddot{\pi}\left(2(\mathfrak{z}-\grave{\mathfrak{y}}), \frac{\breve{\tau}}{2}\right) \\
&= \ddot{\pi}\left(\acute{\mathfrak{e}}-\mathfrak{z}, \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \ddot{\pi}\left(\mathfrak{z}-\grave{\mathfrak{y}}, \frac{\breve{\tau}}{2^{\delta+1}}\right) \\
&\leqslant (1-\mathfrak{r}_5)\diamond(1-\mathfrak{r}_5) \leqslant (1-\mathfrak{r}_4)\diamond(1-\mathfrak{r}_4) \leqslant 1-\mathfrak{r}_0 < \mathfrak{r}_2,
\end{aligned}$$

which is a contradiction. Therefore $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ is Hausdorff. □

In the following, we give further properties of a δ-homogeneous N$\hat{\mathfrak{F}}$M.

DEFINITION 3.3. Let $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ be a δ-homogeneous N$\hat{\mathfrak{F}}$M.
1. A subset $\mathfrak{A}$ of Ξ is said to be $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-bounded if there are $\breve{\tau} > 0$ and $\mathfrak{r} \in (0,1)$ such that for all $\acute{\mathfrak{e}} \in \mathfrak{A}$, $\dot{\lambda}(\acute{\mathfrak{e}}, \breve{\tau}) > 1-\mathfrak{r}$, $\ddot{\pi}(\acute{\mathfrak{e}}, \breve{\tau}) < \mathfrak{r}$ and $\dddot{\varpi}(\acute{\mathfrak{e}}, \breve{\tau}) < \mathfrak{r}$.
2. A subset $\mathfrak{A}$ of Ξ is said to be $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-compact if every $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-open cover of $\mathfrak{A}$ has a finite subcover.
3. A sequence $\{\acute{\mathfrak{e}}_n\}$ in Ξ is said to be $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-convergent to $\acute{\mathfrak{e}} \in \Xi$ if for every $\mathfrak{r} \in (0,1)$ and $\breve{\tau} > 0$ there exists $n_0 \in \mathbb{N}$ such that for each $n > n_0, \acute{\mathfrak{e}}_n \in \mathfrak{B}(\acute{\mathfrak{e}}, \mathfrak{r}, \breve{\tau})$.

EXAMPLE 3.3. Let $\Xi = \mathbb{R}$ and let $\dot{\varrho}, \dot{\lambda}, *$ and $\diamond$ be as in Example (2.2).
(i) Consider $\mathfrak{V} = \{\acute{\mathfrak{e}} \in \mathbb{R} : 0 < \acute{\mathfrak{e}} < 1\}$.
Then $\mathfrak{V}$ is a bounded set in $(\mathbb{R}, \dot{\lambda}, 1-\dot{\lambda}, *, \diamond)$.
(ii) Each finite set in $(\mathbb{R}, \dot{\lambda}, 1-\dot{\lambda}, *, \diamond)$ is $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-compact.
(iii) The sequence $\left\{\frac{1}{n}\right\}$ is $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-convergent to 0 in $(\mathbb{R}, \dot{\lambda}, 1-\dot{\lambda}, *, \diamond)$ by choosing n_0 such that $1-\breve{\tau} < \frac{1}{n_0} < \mathfrak{r}$.

THEOREM 3.3. *Every $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-compact subset of a δ-homogeneous N$\hat{\mathfrak{F}}$M$(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$, is $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-bounded.*

PROOF. Let $\mathfrak{A}$ be a $\dot{\lambda}-\ddot{\pi}-\dddot{\varpi}$-compact subset of $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$.
Fix $\breve{\tau} > 0$ and $\mathfrak{r} \in (0,1)$, then the family $\left\{\mathfrak{B}\left(\acute{\mathfrak{e}}, \mathfrak{r}, \frac{\breve{\tau}}{2^{\delta+1}}\right) : \acute{\mathfrak{e}} \in \mathfrak{A}\right\}$ is an open cover of $\mathfrak{A}$, since $\mathfrak{A}$ is compact there exist $\acute{\mathfrak{e}}_1, \ldots, \acute{\mathfrak{e}}_n \in \mathfrak{A}$ such that $\mathfrak{A} \subset \cup_{i=1}^{n} \mathfrak{B}\left(\acute{\mathfrak{e}}_i, \mathfrak{r}, \frac{\breve{\tau}}{2^{\delta+1}}\right)$. Hence for each $\acute{\mathfrak{e}} \in \mathfrak{A}$ there exists i such that $\acute{\mathfrak{e}} \in \mathfrak{B}\left(\acute{\mathfrak{e}}_i, \mathfrak{r}, \frac{\breve{\tau}}{2^{\delta+1}}\right)$. Thus

$$\dot{\lambda}\left(\acute{\mathfrak{e}}-\acute{\mathfrak{e}}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) > 1-\mathfrak{r}, \ddot{\pi}\left(\acute{\mathfrak{e}}-\acute{\mathfrak{e}}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \mathfrak{r} \text{ and } \dddot{\varpi}\left(\acute{\mathfrak{e}}-\acute{\mathfrak{e}}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \mathfrak{r}.$$

Put $\hat{\epsilon}_1 = \min\left\{\dot{\lambda}\left(\acute{\mathfrak{e}}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) : 1 \leqslant i \leqslant n\right\}$, $\hat{\epsilon}_2 = \max\left\{\ddot{\pi}\left(\acute{\mathfrak{e}}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) : 1 \leqslant i \leqslant n\right\}$, and $\hat{\epsilon}_3 = \max\left\{\dddot{\varpi}\left(\acute{\mathfrak{e}}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) : 1 \leqslant i \leqslant n\right\}$ it is clear that $\hat{\epsilon}_1, \hat{\epsilon}_2, \hat{\epsilon}_3 > 0$.

Hence for some $\breve{\varsigma}_1, \breve{\varsigma}_2, \breve{\varsigma}_3 \in (0,1)$ we have

$$\begin{aligned}\dot{\lambda}(\mathfrak{e}, \breve{\tau}) &= \dot{\lambda}((\mathfrak{e}-\mathfrak{e}_i)+\mathfrak{e}_i, \breve{\tau}) \geqslant \dot{\lambda}\left(2(\mathfrak{e}-\mathfrak{e}_i), \frac{\breve{\tau}}{2}\right) * \dot{\lambda}\left(2\mathfrak{e}_i, \frac{\breve{\tau}}{2}\right)\\ &= \dot{\lambda}\left(\mathfrak{e}-\mathfrak{e}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) * \dot{\lambda}\left(\Xi_I, \frac{\breve{\tau}}{2^{\delta+1}}\right) \geqslant (1-\mathfrak{r}) * \hat{\epsilon}_1 > 1-\breve{\varsigma}_1,\\ \ddot{\pi}(\mathfrak{e}, \breve{\tau}) &= \ddot{\pi}((\mathfrak{e}-\mathfrak{e}_i)+\mathfrak{e}_i, \breve{\tau}) \leqslant \ddot{\pi}\left(2(\mathfrak{e}-\mathfrak{e}_i), \frac{\breve{\tau}}{2}\right) \diamond \ddot{\pi}\left(2\mathfrak{e}_i, \frac{\breve{\tau}}{2}\right)\\ &= \ddot{\pi}\left(\mathfrak{e}-\mathfrak{e}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \ddot{\pi}\left(\Xi_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) \leqslant \mathfrak{r} \diamond \hat{\epsilon}_2 < \breve{\varsigma}_2, \text{ and}\\ \dddot{\varpi}(\mathfrak{e}, \breve{\tau}) &= \dddot{\varpi}((\mathfrak{e}-\mathfrak{e}_i)+\mathfrak{e}_i, \breve{\tau}) \leqslant \dddot{\varpi}\left(2(\mathfrak{e}-\mathfrak{e}_i), \frac{\breve{\tau}}{2}\right) \diamond \dddot{\varpi}\left(2\mathfrak{e}_i, \frac{\breve{\tau}}{2}\right)\\ &= \dddot{\varpi}\left(\mathfrak{e}-\mathfrak{e}_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \dddot{\varpi}\left(\Xi_i, \frac{\breve{\tau}}{2^{\delta+1}}\right) \leqslant \mathfrak{r} \diamond \hat{\epsilon}_3 < \breve{\varsigma}_3.\end{aligned}$$

Taking $\breve{\varsigma} = \max\{\breve{\varsigma}_1, \breve{\varsigma}_2, \breve{\varsigma}_3\}$ we conclude $\dot{\lambda}(\mathfrak{e}, \breve{\tau}) > 1-\breve{\varsigma}$, $\ddot{\pi}(\mathfrak{e}, \breve{\tau}) < \breve{\varsigma}$, and $\dddot{\varpi}(\mathfrak{e}, \breve{\tau}) < \breve{\varsigma}$ consequently $\mathfrak{A}$ is $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-bounded. □

THEOREM 3.4. *Let* $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ *be a* δ*-homogeneous* $N\hat{\mathfrak{F}}M$*and* $\{\mathfrak{e}_n\}$ *a sequence in* Ξ*. Then* $\mathfrak{e}_n \to \mathfrak{e}$ *if and only if* $\dot{\lambda}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 1, \ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 0$ *and* $\dddot{\varpi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 0$.

PROOF. Fix $\breve{\tau} > 0$. Assume that $\mathfrak{e}_n \to \mathfrak{e}$, then for $\mathfrak{r} \in (0,1)$ there exists $n_0 \in \mathbb{N}$ such that for each $n \geqslant n_0$, $\mathfrak{e}_n \in \mathfrak{B}(\mathfrak{e}, \mathfrak{r}, \breve{\tau})$, so $\dot{\lambda}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) > 1-\mathfrak{r}, \ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) < \mathfrak{r}$ and $\dddot{\varpi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) < \mathfrak{r}$.
Hence $\dot{\lambda}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 1, \ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 0,$ and $\dddot{\varpi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 0$.
Conversely, for each $\breve{\tau} > 0$, let $\dot{\lambda}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 1, \ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 0$ and $\dddot{\varpi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 0$. Then for $\mathfrak{r} \in (0,1)$, there exists $n_0 \in \mathbb{N}$ such that for each $n \geqslant n_0$,
$1 - \dot{\lambda}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) < \mathfrak{r}, \ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) < \mathfrak{r}$ and $\dddot{\varpi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) < \mathfrak{r}$.
Therefore $\dot{\lambda}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) > 1-\mathfrak{r}, \ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) < \mathfrak{r}$ and $\dddot{\varpi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) < \mathfrak{r}$ for all $n \geqslant n_0$, that is, $\mathfrak{e}_n \in \mathfrak{B}(\mathfrak{e}, \mathfrak{r}, \breve{\tau})$ and so $\mathfrak{e}_n \to \mathfrak{e}$. □

In the following, we give some related results of completeness of an N$\hat{\mathfrak{F}}$M.

DEFINITION 3.4. Let $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ be an N$\hat{\mathfrak{F}}$M.
1. A sequence $\{\mathfrak{e}_n\}$ in Ξ is called $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-Cauchy if for every $\varepsilon > 0$ and $\breve{\tau} > 0$ there exists $n_0 \in \mathbb{N}$ such that $\dot{\lambda}(\mathfrak{e}_n - \mathfrak{e}_m, \breve{\tau}) > 1-\mathfrak{r}$, $\ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}_m, \breve{\tau}) < \mathfrak{r}$ and $\ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}_m, \breve{\tau}) < \mathfrak{r}$ for all $m, n \geqslant n_0$.
2. Ξ is called $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-complete if every $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-Cauchy sequence is $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-convergent.

THEOREM 3.5. *Let* $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *_{\mathfrak{P}}, \diamond_{\mathfrak{P}}, \odot_{\mathfrak{P}})$ *be a* δ*-homogeneous* $N\hat{\mathfrak{F}}M$*. Then every* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-convergent sequence in* Ξ *is a* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-Cauchy sequence.*

PROOF. Let $\{\mathfrak{e}_n\}$ be $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-convergent to $\mathfrak{e} \in \Xi$.
Then for every $\varepsilon > 0$ and $\breve{\tau} > 0$ there exists $n_0 \in \mathbb{N}$ such that

$\dot{\lambda}\left(\mathfrak{e}_n - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right) > 1-\varepsilon, \ddot{\pi}\left(\mathfrak{e}_n - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \varepsilon$ and $\dddot{\varpi}\left(\mathfrak{e}_n - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \varepsilon$ for all $n \geqslant n_0$. For all $m, n \geqslant n_0$ we get

$$\begin{aligned}
\dot{\lambda}(\mathfrak{e}_m - \mathfrak{e}_n, \breve{\tau}) &\geqslant \dot{\lambda}\left(2(\mathfrak{e}_m - \acute{\mathfrak{e}}), \frac{\breve{\tau}}{2}\right) * \dot{\lambda}\left(2(\mathfrak{e}_n - \acute{\mathfrak{e}}), \frac{\breve{\tau}}{2}\right)\\
&\geqslant \dot{\lambda}\left(\mathfrak{e}_m - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right) * \dot{\lambda}\left(\mathfrak{e}_n - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right)\\
&> (1-\varepsilon) *_{\mathfrak{P}} (1-\varepsilon) = 1-\varepsilon,\\
\ddot{\pi}(\mathfrak{e}_m - \mathfrak{e}_n, \breve{\tau}) &\leqslant \ddot{\pi}\left(2(\mathfrak{e}_m - \acute{\mathfrak{e}}), \frac{\breve{\tau}}{2}\right) \diamond \ddot{\pi}\left(2(\mathfrak{e}_n - \acute{\mathfrak{e}}), \frac{\breve{\tau}}{2}\right)\\
&\leqslant \ddot{\pi}\left(\mathfrak{e}_m - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \ddot{\pi}\left(\mathfrak{e}_n - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right)\\
&< \varepsilon \diamond_{\mathfrak{P}} \varepsilon = \varepsilon \text{ and}\\
\dddot{\varpi}(\mathfrak{e}_m - \mathfrak{e}_n, \breve{\tau}) &\leqslant \dddot{\varpi}\left(2(\mathfrak{e}_m - \acute{\mathfrak{e}}), \frac{\breve{\tau}}{2}\right) \odot \dddot{\varpi}\left(2(\mathfrak{e}_n - \acute{\mathfrak{e}}), \frac{\breve{\tau}}{2}\right)\\
&\leqslant \dddot{\varpi}\left(\mathfrak{e}_m - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right) \odot \dddot{\varpi}\left(\mathfrak{e}_n - \acute{\mathfrak{e}}, \frac{\breve{\tau}}{2^{\delta+1}}\right)\\
&< \varepsilon \odot_{\mathfrak{P}} \varepsilon = \varepsilon
\end{aligned}$$

□

REMARK 3.2. (1) Theorem (3.5) shows that in an N$\hat{\mathfrak{F}}$M, a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-convergent sequence is not necessarily a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-Cauchy sequence, and the δ-homogeneity and the choice of t-norm and t-conorm are essential.
(2) From Definition (3.4), it is clear that each $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-closed subspace of $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-complete $\hat{\mathfrak{F}}$-modular space is $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-complete.

THEOREM 3.6. *Let* $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ *be a* δ*-homogeneous N*$\hat{\mathfrak{F}}$*M and* $\mathfrak{Y}$ *a subset of* Ξ. *If every* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-Cauchy sequence of* $\mathfrak{Y}$ *is* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-convergent in* Ξ, *then every* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-Cauchy sequence of* $\bar{\mathfrak{Y}}$ *is* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-convergent in* Ξ, *where* $\bar{\mathfrak{Y}}$ *denotes the* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-closure of* $\mathfrak{Y}$.

PROOF. Let $\{\mathfrak{e}_n\}$ be a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-Cauchy sequence of $\bar{\mathfrak{Y}}$, then for each $n \in \mathbb{N}$ and $\breve{\tau} > 0$, there exists $\mathfrak{y}_n \in \mathfrak{Y}$ such that
$\dot{\lambda}\left(\mathfrak{e}_n - \mathfrak{y}_n, \frac{\breve{\tau}}{4^{\delta+1}}\right) > 1 - \frac{1}{n+1}$ $\ddot{\pi}\left(\mathfrak{e}_n - \mathfrak{y}_n, \frac{\breve{\tau}}{4^{\delta+1}}\right) < \frac{1}{n+1}$ and $\dddot{\varpi}\left(\mathfrak{e}_n - \mathfrak{y}_n, \frac{\breve{\tau}}{4^{\delta+1}}\right) < \frac{1}{n+1}$.
Since $\dot{\lambda}(\acute{\mathfrak{e}}, .)$ is non-decreasing and $\ddot{\pi}(\acute{\mathfrak{e}}, .)$ is non-increasing, we have
$\dot{\lambda}\left(\mathfrak{e}_n - \mathfrak{y}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) > 1 - \frac{1}{n+1}$ $\ddot{\pi}\left(\mathfrak{e}_n - \mathfrak{y}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \frac{1}{n+1}$ and $\dddot{\varpi}\left(\mathfrak{e}_n - \mathfrak{y}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \frac{1}{n+1}$.
Moreover for each $\mathfrak{r} \in (0,1)$ and $\breve{\tau} > 0$, there exists $n_0 \in \mathbb{N}$ such that
$\dot{\lambda}\left(\mathfrak{e}_n - \mathfrak{e}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) > 1 - \mathfrak{r}$, $\ddot{\pi}\left(\mathfrak{e}_n - \mathfrak{e}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) < \mathfrak{r}$ and $\dddot{\varpi}\left(\mathfrak{e}_n - \mathfrak{e}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) < \mathfrak{r}$
for all $m, n \geqslant n_0$.
That is, $\dot{\lambda}\left(\mathfrak{e}_n - \mathfrak{e}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) \to 1$, $\ddot{\pi}\left(\mathfrak{e}_n - \mathfrak{e}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) \to 0$ and $\dddot{\varpi}\left(\mathfrak{e}_n - \mathfrak{e}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) \to 0$.
Now we show that $\{\mathfrak{y}_n\}$ is a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-Cauchy sequence in $\mathfrak{Y}$.

For all $m, n \geqslant n_0$ we have

$$\begin{aligned}\dot{\lambda}(\grave{\mathfrak{y}}_n - \grave{\mathfrak{y}}_m, \breve{\tau}) &\geqslant \dot{\lambda}\left(2(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n), \frac{\breve{\tau}}{2}\right) * \dot{\lambda}\left(2(\acute{\mathfrak{e}}_n - \grave{\mathfrak{y}}_n), \frac{\breve{\tau}}{2}\right)\\ &\geqslant \dot{\lambda}\left(2(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n), \frac{\breve{\tau}}{2}\right) * \dot{\lambda}\left(4(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}_m), \frac{\breve{\tau}}{4}\right) * \dot{\lambda}\left(4(\acute{\mathfrak{e}}_m - \grave{\mathfrak{y}}_m), \frac{\breve{\tau}}{4}\right)\\ &= \dot{\lambda}\left(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) * \dot{\lambda}\left(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) * \dot{\lambda}\left(\acute{\mathfrak{e}}_m - \grave{\mathfrak{y}}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right)\\ &> \left(1 - \frac{1}{n+1}\right) * (1-r) * \left(1 - \frac{1}{m+1}\right).\end{aligned}$$

Since $*$ is continuous $\dot{\lambda}(\grave{\mathfrak{y}}_n - \grave{\mathfrak{y}}_m, \breve{\tau}) \to 1$, Furthermore

$$\begin{aligned}\ddot{\pi}(\grave{\mathfrak{y}}_n - \grave{\mathfrak{y}}_m, \breve{\tau}) &\leqslant \ddot{\pi}\left(2(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n), \frac{\breve{\tau}}{2}\right) \diamond \ddot{\pi}\left(2(\acute{\mathfrak{e}}_n - \grave{\mathfrak{y}}_n), \frac{\breve{\tau}}{2}\right)\\ &\leqslant \ddot{\pi}\left(2(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n), \frac{\breve{\tau}}{2}\right) \diamond \ddot{\pi}\left(4(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}_m), \frac{\breve{\tau}}{4}\right) \diamond \ddot{\pi}\left(4(\acute{\mathfrak{e}}_m - \grave{\mathfrak{y}}_m), \frac{\breve{\tau}}{4}\right)\\ &= \ddot{\pi}\left(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \ddot{\pi}\left(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) \diamond \ddot{\pi}\left(\acute{\mathfrak{e}}_m - \grave{\mathfrak{y}}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right)\\ &< \frac{1}{n+1} \diamond \mathfrak{r} \diamond \frac{1}{m+1}, \text{ and}\\ \dddot{\varpi}(\grave{\mathfrak{y}}_n - \grave{\mathfrak{y}}_m, \breve{\tau}) &\leqslant \dddot{\varpi}\left(2(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n), \frac{\breve{\tau}}{2}\right) \odot \dddot{\varpi}\left(2(\acute{\mathfrak{e}}_n - \grave{\mathfrak{y}}_n), \frac{\breve{\tau}}{2}\right)\\ &\leqslant \dddot{\varpi}\left(2(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n), \frac{\breve{\tau}}{2}\right) \odot \dddot{\varpi}\left(4(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}_m), \frac{\breve{\tau}}{4}\right) \odot \dddot{\varpi}\left(4(\acute{\mathfrak{e}}_m - \grave{\mathfrak{y}}_m), \frac{\breve{\tau}}{4}\right)\\ &= \dddot{\varpi}\left(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) \odot \dddot{\varpi}\left(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right) \odot \dddot{\varpi}\left(\acute{\mathfrak{e}}_m - \grave{\mathfrak{y}}_m, \frac{\breve{\tau}}{4^{\delta+1}}\right)\\ &< \frac{1}{n+1} \odot \mathfrak{r} \odot \frac{1}{m+1}.\end{aligned}$$

Hence $\ddot{\pi}(\grave{\mathfrak{y}}_n - \grave{\mathfrak{y}}_m, \breve{\tau}) \to 0$, that is, $\{\grave{\mathfrak{y}}_n\}$ is Cauchy in $\mathfrak{Y}$, so it is $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-convergent to $\acute{\mathfrak{e}} \in \mathfrak{X}$. Thus for each $\varepsilon > 0$ and $\breve{\tau} > 0$ there exists $n_1 \in \mathbb{N}$ such that
$\dot{\lambda}\left(\acute{\mathfrak{e}} - \grave{\mathfrak{y}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) > 1 - \varepsilon, \ddot{\pi}\left(\acute{\mathfrak{e}} - \grave{\mathfrak{y}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \varepsilon$ and $\dddot{\varpi}\left(\acute{\mathfrak{e}} - \grave{\mathfrak{y}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \varepsilon$
for all $n \geqslant n_1$. Therefore

$$\begin{aligned}\dot{\lambda}(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}, \breve{\tau}) &\geqslant \dot{\lambda}\left(2(\acute{\mathfrak{e}}_n - \grave{\mathfrak{y}}_n), \frac{\breve{\tau}}{2}\right) * \dot{\lambda}\left(2(\grave{\mathfrak{y}}_n - \acute{\mathfrak{e}}_n), \frac{\breve{\tau}}{2}\right)\\ &= \dot{\lambda}\left(\acute{\mathfrak{e}}_n - \grave{\mathfrak{y}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) * \dot{\lambda}\left(\acute{\mathfrak{e}}_n - \grave{\mathfrak{y}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right)\\ &> (1-\varepsilon) * \left(1 - \frac{1}{n+1}\right),\end{aligned}$$

consequently, $\dot{\lambda}(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}, \breve{\tau}) \to 1$. Similarly we have
$\ddot{\pi}(\acute{\mathfrak{e}}_n - \acute{\mathfrak{e}}, \breve{\tau}) \leqslant \ddot{\pi}\left(\acute{\mathfrak{e}}_n - \grave{\mathfrak{y}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \ddot{\pi}\left(\acute{\mathfrak{e}}_n - \grave{\mathfrak{y}}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \varepsilon * \frac{1}{n+1}$,

$\dddot{\varpi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \leqslant \dddot{\varpi}\left(\mathfrak{e}_n - \mathfrak{y}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \dddot{\varpi}\left(\mathfrak{e}_n - \mathfrak{y}_n, \frac{\breve{\tau}}{2^{\delta+1}}\right) < \varepsilon * \frac{1}{n+1}$.
Hence $\ddot{\pi}(\mathfrak{e}_n - \mathfrak{e}, \breve{\tau}) \to 0$, and so the Cauchy sequence $\{\mathfrak{e}_n\}$ in $\mathfrak{Y}$ converges to $\mathfrak{e} \in \Xi$. This completes the proof. □

From Theorem (3.6) we get the following result.

COROLLARY 3.1. *Let* $(\Xi, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ *be a* δ*-homogeneous* $N\hat{\mathfrak{F}}M$ *and let* $\mathfrak{Y}$ *be a dense subset of* Ξ. *If every* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-Cauchy sequence of* $\mathfrak{Y}$ *is* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-convergent in* Ξ, *then* Ξ *is* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-complete. Now we extend the well-known Baire's theorem to* δ*-homogeneous* $N\hat{\mathfrak{F}}M$.

THEOREM 3.7. *Let* $\{\mathfrak{U}_n\}_{n\in\mathbb{N}}$ *be a sequence of* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-dense open subsets in* δ*-homogeneous neutrosophic* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*- complete* $\hat{\mathfrak{F}}$*-modular space* $(\Xi, \dot{\lambda}, \ddot{\pi}, *_{\mathfrak{P}}, \diamond_{\mathfrak{P}})$. *Then* $\bigcap_{n=1}^{\infty} \mathfrak{U}_n$ *is* $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$*-dense in* Ξ.

PROOF. Consider the $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-ball $\mathfrak{B}(\mathfrak{e}, \mathfrak{r}, \breve{\tau})$ and let $\mathfrak{y} \in \mathfrak{B}(\mathfrak{e}, \mathfrak{r}, \breve{\tau})$. Then $\dot{\lambda}(\mathfrak{e} - \mathfrak{y}, 2\breve{\tau}) > 1 - \mathfrak{r}$ and $\ddot{\pi}(\mathfrak{e} - \mathfrak{y}, 2\breve{\tau}) < \mathfrak{r}$.
Since $\dot{\lambda}(\mathfrak{e} - \mathfrak{y}, .)$ and $\ddot{\pi}(\mathfrak{e} - \mathfrak{y}, .)$ are continuous, there exists $\xi_{\mathfrak{y}} > 0$ such that $\dot{\lambda}\left(\mathfrak{e} - \mathfrak{y}, \frac{\breve{\tau}-\varepsilon}{2^{\delta-1}}\right) > 1 - \mathfrak{r}$ and $\ddot{\pi}\left(\mathfrak{e} - \mathfrak{y}, \frac{\breve{\tau}-\varepsilon}{2^{\delta-1}}\right) < \mathfrak{r}$ for some $\varepsilon > 0$ with $\frac{\breve{\tau}-\varepsilon}{2^{\delta-1}} > 0$ and $\frac{\varepsilon}{2^{\delta-1}} \in (0, \xi_{\mathfrak{y}})$.
We claim that $\overline{\mathfrak{B}\left(\mathfrak{y}, \mathfrak{r}', \frac{\varepsilon}{4^\delta}\right)} \subseteq \mathfrak{B}(\mathfrak{e}, \mathfrak{r}, 2t)$.
Choose $\mathfrak{r}' \in (0, 1)$ and $\mathfrak{z} \in \overline{\mathfrak{B}\left(\mathfrak{y}, \mathfrak{r}', \frac{\varepsilon}{4^\delta}\right)}$, then there exists a sequence $\{\mathfrak{z}_n\}$ in $\overline{\mathfrak{B}\left(\mathfrak{y}, \mathfrak{r}', \frac{\varepsilon}{4^\delta}\right)}$ which is $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-converges to $\mathfrak{z}$, so we have

$$\begin{aligned}
\dot{\lambda}\left(\mathfrak{z} - \mathfrak{y}, \frac{\varepsilon}{2^{\delta-1}}\right) &\geqslant \dot{\lambda}\left(2(\mathfrak{z} - \mathfrak{z}_n), \frac{\varepsilon}{2^\delta}\right) * \mathfrak{M}\dot{\lambda}\left(2(\mathfrak{z}_n - \mathfrak{y}), \frac{\varepsilon}{2^\delta}\right) \\
&= \dot{\lambda}\left(z - \mathfrak{z}_n, \frac{\varepsilon}{4^\delta}\right) * \mathfrak{M}\dot{\lambda}\left(\mathfrak{z}_n - \mathfrak{y}, \frac{\varepsilon}{4^\delta}\right) > 1 - \mathfrak{r}, \\
\ddot{\pi}\left(\mathfrak{z} - \mathfrak{y}, \frac{\varepsilon}{2^{\delta-1}}\right) &\leqslant \ddot{\pi}\left(2(\mathfrak{z} - \mathfrak{z}_n), \frac{\varepsilon}{2^\delta}\right) \diamond_{\mathfrak{P}} \ddot{\pi}\left(2(\mathfrak{z}_n - \mathfrak{y}), \frac{\varepsilon}{2^\delta}\right) \\
&= \ddot{\pi}\left(\mathfrak{z} - \mathfrak{z}_n, \frac{\varepsilon}{4^\delta}\right) \diamond_{\mathfrak{P}} \ddot{\pi}\left(\mathfrak{z}_n - \mathfrak{y}, \frac{\varepsilon}{4^\delta}\right) < \mathfrak{r}, \text{ and} \\
\dddot{\varpi}\left(\mathfrak{z} - \mathfrak{y}, \frac{\varepsilon}{2^{\delta-1}}\right) &\leqslant \dddot{\varpi}\left(2(\mathfrak{z} - \mathfrak{z}_n), \frac{\varepsilon}{2^\delta}\right) \odot_{\mathfrak{P}} \dddot{\varpi}\left(2(\mathfrak{z}_n - \mathfrak{y}), \frac{\varepsilon}{2^\delta}\right) \\
&= \dddot{\varpi}\left(\mathfrak{z} - \mathfrak{z}_n, \frac{\varepsilon}{4^\delta}\right) \odot_{\mathfrak{P}} \dddot{\varpi}\left(\mathfrak{z}_n - \mathfrak{y}, \frac{\varepsilon}{4^\delta}\right) < \mathfrak{r}
\end{aligned}$$

Therefore we have

$$\begin{aligned}
\dot{\lambda}(\mathfrak{e} - \mathfrak{z}, 2\breve{\tau}) &= \dot{\lambda}(2(\mathfrak{z} - \mathfrak{y}), 2\varepsilon) * \mathfrak{M}\dot{\lambda}(2(\mathfrak{e} - \mathfrak{y}), 2(\breve{\tau} - \varepsilon)) \\
&= \dot{\lambda}\left(\mathfrak{z} - \mathfrak{y}, \frac{\varepsilon}{2^{\delta-1}}\right) * \mathfrak{M}\dot{\lambda}\left(\mathfrak{e} - \mathfrak{y}, \frac{\breve{\tau} - \varepsilon}{2^{\delta-1}}\right) \\
&\geqslant (1 - \mathfrak{r}) *_{\mathfrak{P}} (1 - \mathfrak{r}) = 1 - \mathfrak{r},
\end{aligned}$$

$$\begin{aligned}
\ddot{\pi}(\acute{\mathfrak{e}}-\mathfrak{z},2\breve{\tau}) &= \dot{\lambda}(2(\mathfrak{z}-\grave{\mathfrak{y}}),2\varepsilon)\diamond_{\mathfrak{P}}\ddot{\pi}(2(\acute{\mathfrak{e}}-\grave{\mathfrak{y}}),2(\breve{\tau}-\varepsilon))\\
&= \ddot{\pi}\left(\mathfrak{z}-\grave{\mathfrak{y}},\frac{\varepsilon}{2^{\delta-1}}\right)\diamond_{\mathfrak{P}}\ddot{\pi}\left(\acute{\mathfrak{e}}-\grave{\mathfrak{y}},\frac{\breve{\tau}-\varepsilon}{2^{\delta-1}}\right)\\
&\leqslant \mathfrak{r}\diamond_{\mathfrak{P}}\mathfrak{r}=\mathfrak{r}, \text{and}\\
\ddot{\varpi}(\acute{\mathfrak{e}}-\mathfrak{z},2\breve{\tau}) &= \dot{\lambda}(2(\mathfrak{z}-\grave{\mathfrak{y}}),2\varepsilon)\odot_{\mathfrak{P}}\ddot{\varpi}(2(\acute{\mathfrak{e}}-\grave{\mathfrak{y}}),2(\breve{\tau}-\varepsilon))\\
&= \ddot{\varpi}\left(\mathfrak{z}-\grave{\mathfrak{y}},\frac{\varepsilon}{2^{\delta-1}}\right)\odot_{\mathfrak{P}}\ddot{\varpi}\left(\acute{\mathfrak{e}}-\grave{\mathfrak{y}},\frac{\breve{\tau}-\varepsilon}{2^{\delta-1}}\right)\\
&\leqslant \mathfrak{r}\odot_{\mathfrak{P}}\mathfrak{r}=\mathfrak{r}.
\end{aligned}$$

So the claim is true and hence if V is a nonempty $\dot{\lambda}-\ddot{\pi}-\ddot{\varpi}$-open set of Ξ, then $V\bigcap U_1$ is nonempty and $\dot{\lambda}-\ddot{\pi}-\ddot{\varpi}$-open.
Suppose $\acute{\mathfrak{e}}_1\in V\bigcap U_1$, so there exist $\mathfrak{r}_1\in(0,1)$ and $\breve{\tau}_1>0$ such that $\mathfrak{B}\left(\acute{\mathfrak{e}}_1,\mathfrak{r}_1,\frac{\breve{\tau}_1}{2^{\delta-1}}\right)\subseteq V\bigcap U_1$.
Choose $r_1'<\mathfrak{r}_1$ and $\breve{\tau}_1'=\min\{\breve{\tau}_1,1\}$ such that $\mathfrak{B}\left(\acute{\mathfrak{e}}_1,\mathfrak{r}',\frac{\breve{\tau}_1'}{2^{\delta-1}}\right)\subseteq V\bigcap U_1$.
Since U_2 is $\dot{\lambda}-\ddot{\pi}-\ddot{\varpi}$-dense in Ξ, we have $\mathfrak{B}\left(\acute{\mathfrak{e}}_1,\mathfrak{r}_1',\frac{\breve{\tau}_1'}{2^{\delta-1}}\right)\bigcap U_2\neq\emptyset$.
Let $\acute{\mathfrak{e}}_2\in\mathfrak{B}\left(\acute{\mathfrak{e}}_1,\mathfrak{r}_1',\frac{\breve{\tau}_1'}{2^{\delta-1}}\right)\bigcap U_2$, hence there exist $\mathfrak{r}_2\in\left(0,\frac{1}{2}\right)$ and $\breve{\tau}_2>0$ such that $\mathfrak{B}\left(\acute{\mathfrak{e}}_2,\mathfrak{r}_2,\frac{\breve{\tau}_2}{2^{\delta-1}}\right)\subseteq\mathfrak{B}\left(\acute{\mathfrak{e}}_1,\mathfrak{r}_1',\frac{\breve{\tau}_1'}{2^{\delta-1}}\right)\bigcap U_2$.
Choose $\mathfrak{r}_2'<\mathfrak{r}_2$ and $\breve{\tau}_2'=\min\left\{\breve{\tau}_2,\frac{1}{2}\right\}$ such that $\overline{\mathfrak{B}\left(\acute{\mathfrak{e}}_2,\mathfrak{r}_2',\frac{t\breve{\tau}_2'}{2^{\delta-1}}\right)}\subseteq V\bigcap U_2$.
By induction, we can obtain a sequence $\{\acute{\mathfrak{e}}_n\}$ in Ξ and two sequences $\{\mathfrak{r}_n'\},\{\breve{\tau}_n'\}$ such that $0<\mathfrak{r}_n'<\frac{1}{n},0<\breve{\tau}_n'<\frac{1}{n}$ and $\overline{\mathfrak{B}\left(\acute{\mathfrak{e}}_n,\mathfrak{r}_n',\frac{\breve{\tau}_n'}{2^{\delta-1}}\right)}\subseteq V\bigcap U_n$.
We show that $\{\acute{\mathfrak{e}}_n\}$ is $\dot{\lambda}-\ddot{\pi}-\ddot{\varpi}$-Cauchy.
Get $\breve{\tau}>0$ and $\mathfrak{r}\in(0,1)$, then we can choose $\mathfrak{k}\in\mathbb{N}$ such that $2\breve{\tau}_k'<\breve{\tau}$ and $\mathfrak{r}_{\mathfrak{k}}'<\mathfrak{r}$.
Since $\acute{\mathfrak{e}}_m,\acute{\mathfrak{e}}_n\in\mathfrak{B}\left(\acute{\mathfrak{e}}_{\mathfrak{k}},\mathfrak{r}_{\mathfrak{k}}',\frac{\breve{\tau}_{\mathfrak{k}}'}{2^{\delta-1}}\right)$, for $m,n\geqslant\mathfrak{k}$, we get

$$\begin{aligned}
\dot{\lambda}(\acute{\mathfrak{e}}_m-\acute{\mathfrak{e}}_n,2\breve{\tau}) &\geqslant \dot{\lambda}(\acute{\mathfrak{e}}_m-\acute{\mathfrak{e}}_n,4\breve{\tau}_{\mathfrak{k}}')\\
&\geqslant \dot{\lambda}\left(2(\acute{\mathfrak{e}}_m-\acute{\mathfrak{e}}_{\mathfrak{k}}),2\breve{\tau}_{\mathfrak{k}}'\right)*\mathfrak{M}\dot{\lambda}(2(\acute{\mathfrak{e}}_{\mathfrak{k}}-\acute{\mathfrak{e}}_n),2\breve{\tau}_{\mathfrak{k}}')\\
&= \dot{\lambda}\left(\acute{\mathfrak{e}}_m-\acute{\mathfrak{e}}_{\mathfrak{k}},\frac{\breve{\tau}_{\mathfrak{k}}'}{2^{\delta-1}}\right)*\mathfrak{M}\dot{\lambda}\left(\acute{\mathfrak{e}}_{\mathfrak{k}}-\acute{\mathfrak{e}}_n,\frac{\breve{\tau}_{\mathfrak{k}}'}{2^{\delta-1}}\right)\\
&\geqslant (1-\mathfrak{r}_{\mathfrak{k}})*_{\mathfrak{P}}(1-\mathfrak{r}_{\mathfrak{k}})>1-\mathfrak{r},
\end{aligned}$$

$$\begin{aligned}
\ddot{\pi}(\acute{\mathfrak{e}}_m - \acute{\mathfrak{e}}_n, 2\breve{\tau}) &\leqslant \ddot{\pi}(\acute{\mathfrak{e}}_m - \acute{\mathfrak{e}}_n, 4\breve{\tau}_{\mathfrak{k}}^{'}) \\
&\leqslant \ddot{\pi}\left(2(\acute{\mathfrak{e}}_m - \acute{\mathfrak{e}}_{\mathfrak{k}}), 2\breve{\tau}_{\mathfrak{k}}^{'}\right) \diamond \mathfrak{M}\ddot{\pi}(2(\acute{\mathfrak{e}}_{\mathfrak{k}} - \acute{\mathfrak{e}}_n), 2\breve{\tau}_{\mathfrak{k}}^{'}) \\
&= \ddot{\pi}\left(\acute{\mathfrak{e}}_m - \acute{\mathfrak{e}}_{\mathfrak{k}}, \frac{\breve{\tau}_{\mathfrak{k}}^{'}}{2^{\delta-1}}\right) \diamond \mathfrak{M}\ddot{\pi}\left(\acute{\mathfrak{e}}_{\mathfrak{k}} - \acute{\mathfrak{e}}_n, \frac{\breve{\tau}_{\mathfrak{k}}^{'}}{2^{\delta-1}}\right) \\
&\leqslant \mathfrak{r}_{\mathfrak{k}}) \diamond_{\mathfrak{P}} \mathfrak{r}_{\mathfrak{k}}) < \mathfrak{r}, \text{ and} \\
\dddot{\varpi}(\acute{\mathfrak{e}}_m - \acute{\mathfrak{e}}_n, 2\breve{\tau}) &\leqslant \dddot{\varpi}(\acute{\mathfrak{e}}_m - \acute{\mathfrak{e}}_n, 4\breve{\tau}_{\mathfrak{k}}^{'}) \\
&\leqslant \dddot{\varpi}\left(2(\acute{\mathfrak{e}}_m - \acute{\mathfrak{e}}_{\mathfrak{k}}), 2\breve{\tau}_{\mathfrak{k}}^{'}\right) \odot \mathfrak{M}\dddot{\varpi}(2(\acute{\mathfrak{e}}_{\mathfrak{k}} - \acute{\mathfrak{e}}_n), 2\breve{\tau}_{\mathfrak{k}}^{'}) \\
&= \dddot{\varpi}\left(\acute{\mathfrak{e}}_m - \acute{\mathfrak{e}}_{\mathfrak{k}}, \frac{\breve{\tau}_{\mathfrak{k}}^{'}}{2^{\delta-1}}\right) \odot \mathfrak{M}\dddot{\varpi}\left(\acute{\mathfrak{e}}_{\mathfrak{k}} - \acute{\mathfrak{e}}_n, \frac{\breve{\tau}_{\mathfrak{k}}^{'}}{2^{\delta-1}}\right) \\
&\leqslant \mathfrak{r}_{\mathfrak{k}}) \odot_{\mathfrak{P}} \mathfrak{r}_{\mathfrak{k}}) < \mathfrak{r}.
\end{aligned}$$

Therefore $\{\acute{\mathfrak{e}}_n\}$ is a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-Cauchy sequence. Since Ξ is $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-complete, there exists $\acute{\mathfrak{e}} \in \Xi$ such that $\acute{\mathfrak{e}}_n \to \acute{\mathfrak{e}}$. For all $n \geqslant \mathfrak{k}, \acute{\mathfrak{e}}_n \in \mathfrak{B}\left(\acute{\mathfrak{e}}_{\mathfrak{k}}, \mathfrak{r}_{\mathfrak{k}}^{'}, \frac{\breve{\tau}_{\mathfrak{k}}^{'}}{2^{\delta-1}}\right)$ and hence $\acute{\mathfrak{e}} \in \overline{\mathfrak{B}\left(\acute{\mathfrak{e}}_{\mathfrak{k}}, \mathfrak{r}_{\mathfrak{k}}^{'}, \frac{\breve{\tau}_{\mathfrak{k}}^{'}}{2^{\delta-1}}\right)} \subseteq V \bigcap U_{\mathfrak{k}}$. This implies that $V \bigcap (\bigcap_{n=1}^{\infty} U_n) \neq \emptyset$.
Therefore $\bigcap_{n=1}^{\infty} U_n$ is $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-dense in Ξ.
Finally, we give the uniform limit theorem in δ-homogeneous N$\hat{\mathfrak{F}}$M. Let Ξ be a nonempty set and let $(\mathfrak{Y}, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$ be an N$\hat{\mathfrak{F}}$M. A sequence $\{\mathfrak{f}_n\}$ of mappings from Ξ to $\mathfrak{Y}$ is called $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-converges uniformly to a mapping $\mathfrak{f} : \Xi \to \mathfrak{Y}$ if, for $\breve{\tau} > 0$ and $\mathfrak{r} \in (0, 1)$, there exists $n_0 \in \mathbb{N}$ such that
$\dot{\lambda}(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}), \breve{\tau}) > 1 - \mathfrak{r}$, $\ddot{\pi}(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}), \breve{\tau}) < \mathfrak{r}$ and $\dddot{\varpi}(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}), \breve{\tau}) < \mathfrak{r}$
for all $n \geqslant n_0$ and $\acute{\mathfrak{e}} \in \Xi$. □

THEOREM 3.8. *Let $\{\mathfrak{f}_n\}$ be a sequence of continuous mappings from a topological space Ξ to a δ-homogeneous N$\hat{\mathfrak{F}}$M $(\mathfrak{Y}, \dot{\lambda}, \ddot{\pi}, \dddot{\varpi}, *, \diamond, \odot)$. If $\{\mathfrak{f}_n\}\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-convergent uniformly to $\mathfrak{f} : \Xi \to \mathfrak{Y}$, then $\mathfrak{f}$ is continuous.*

PROOF. Let V be a $\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$-open set of $\mathfrak{Y}$ and $\acute{\mathfrak{e}}_0 \in \mathfrak{f}^{-1}(V)$, so there exist $\mathfrak{t} > 0$ and $\mathfrak{r} \in (0, 1)$ such that $\mathfrak{B}(\mathfrak{f}(\acute{\mathfrak{e}}_0), \mathfrak{r}, \breve{\tau}) \subset V$.
For $\mathfrak{r} \in (0, 1)$, we can choose $\varsigma \in (0, 1)$ such that $*(1 - \varsigma) * (1 - \varsigma) > 1 - \mathfrak{r}$.
Since $\{\mathfrak{f}_n\}\dot{\lambda} - \ddot{\pi} - \dddot{\varpi}$- converges uniformly to $\mathfrak{f}$, for $\varsigma \in (0, 1)$ and $\breve{\tau} > 0$ there exists $n_0 \in \mathbb{N}$ such that $\dot{\lambda}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{4^{\delta+1}}\right) > 1 - \varsigma$ and $\ddot{\pi}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{4^{\delta+1}}\right) < \varsigma$ for all $n \geqslant n_0$ and $\acute{\mathfrak{e}} \in \Xi$.
Furthermore, each $\mathfrak{f}_n$ is continuous. Then there exists a neighborhood U of $\acute{\mathfrak{e}}_0$ such that $\mathfrak{f}_n(U) \subset \mathfrak{B}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}_0), \varsigma, \frac{\breve{\tau}}{4^{\delta+1}}\right)$.
Therefore $\dot{\lambda}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}_0), \frac{\breve{\tau}}{4^{\delta+1}}\right) > 1 - \varsigma$ and $\ddot{\pi}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}_0), \frac{\breve{\tau}}{4^{\delta+1}}\right) < \varsigma$ for all $n \geqslant n_0$ and $\acute{\mathfrak{e}} \in U$ and so we have

$$\begin{aligned}
&\dot{\lambda}(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0), \breve{\tau}) \geqslant \dot{\lambda}\left(2(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}})), \tfrac{\breve{\tau}}{2}\right) * \dot{\lambda}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \tfrac{\breve{\tau}}{2}\right) \\
&= \dot{\lambda}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \tfrac{\breve{\tau}}{2^{\delta+1}}\right) * \dot{\lambda}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \tfrac{\breve{\tau}}{2^{\delta+1}}\right) \\
&\geqslant \dot{\lambda}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \tfrac{\breve{\tau}}{2^{\delta+1}}\right) * \dot{\lambda}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0)), \tfrac{\breve{\tau}}{2^{\delta+2}}\right) * \dot{\lambda}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}_0) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \tfrac{\breve{\tau}}{2^{\delta+2}}\right)
\end{aligned}$$

$= \dot{\lambda}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{2^{\delta+1}}\right) * \dot{\lambda}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0), \frac{\breve{\tau}}{4^{\delta+1}}\right) * \dot{\lambda}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}_0) - \mathfrak{f}(\acute{\mathfrak{e}}_0), \frac{\breve{\tau}}{4^{\delta+1}}\right)$
$\geqslant (1-\breve{\zeta}) * (1-\breve{\zeta}) * (1-\breve{\zeta}) > 1-\mathfrak{r}$,

$\ddot{\pi}(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0), \breve{\tau}) \leqslant \ddot{\pi}\left(2(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}})), \frac{\breve{\tau}}{2}\right) \diamond \ddot{\pi}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \frac{\breve{\tau}}{2}\right)$
$= \ddot{\pi}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \ddot{\pi}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \frac{\breve{\tau}}{2^{\delta+1}}\right)$
$\leqslant \ddot{\pi}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \ddot{\pi}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0)), \frac{\breve{\tau}}{2^{\delta+2}}\right) \diamond \ddot{\pi}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}_0) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \frac{\breve{\tau}}{2^{\delta+2}}\right)$
$= \ddot{\pi}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{2^{\delta+1}}\right) \diamond \ddot{\pi}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0), \frac{\breve{\tau}}{4^{\delta+1}}\right) \diamond \ddot{\pi}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}_0) - \mathfrak{f}(\acute{\mathfrak{e}}_0), \frac{\breve{\tau}}{4^{\delta+1}}\right)$
$\leqslant (1-\breve{\zeta}) \diamond (1-\breve{\zeta}) \diamond (1-\breve{\zeta}) > 1-\mathfrak{r}$ and

$\dddot{\varpi}(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0), \breve{\tau}) \leqslant \dddot{\varpi}\left(2(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}})), \frac{\breve{\tau}}{2}\right) \odot \dddot{\varpi}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \frac{\breve{\tau}}{2}\right)$
$= \dddot{\varpi}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{2^{\delta+1}}\right) \odot \dddot{\varpi}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \frac{\breve{\tau}}{2^{\delta+1}}\right)$
$\leqslant \dddot{\varpi}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{2^{\delta+1}}\right) \odot \dddot{\varpi}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0)), \frac{\breve{\tau}}{2^{\delta+2}}\right) \odot \dddot{\varpi}\left(2(\mathfrak{f}_n(\acute{\mathfrak{e}}_0) - \mathfrak{f}(\acute{\mathfrak{e}}_0)), \frac{\breve{\tau}}{2^{\delta+2}}\right)$
$= \dddot{\varpi}\left(\mathfrak{f}(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}), \frac{\breve{\tau}}{2^{\delta+1}}\right) \odot \dddot{\varpi}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}) - \mathfrak{f}_n(\acute{\mathfrak{e}}_0), \frac{\breve{\tau}}{4^{\delta+1}}\right) \odot \dddot{\varpi}\left(\mathfrak{f}_n(\acute{\mathfrak{e}}_0) - \mathfrak{f}(\acute{\mathfrak{e}}_0), \frac{\breve{\tau}}{4^{\delta+1}}\right)$
$\leqslant (1-\breve{\zeta}) \odot (1-\breve{\zeta}) \odot (1-\breve{\zeta}) > 1-\mathfrak{r}$
This implies that $\mathfrak{f}(\acute{\mathfrak{e}}) \in \mathfrak{B}(\mathfrak{f}(\acute{\mathfrak{e}}_0), \mathfrak{r}, \breve{\tau}) \subset V$, therefore $\mathfrak{f}(U) \subseteq V$, hence $\mathfrak{f}$ is continuous. □

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