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ENCRYPTION AND DECRYPTION ALGORITHMS ON CONTROLLED METRIC TYPE SPACES

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ABSTRACT. In this paper, we introduce the encryption algorithm and the decryption algorithm on controlled metric type spaces. Our algorithms is based on using the control function. To do this, we construct the character table and give two algorithms for encryption and decryption. Finally, we support our algorithms with different examples.

1. Introduction and History

Metric space structure is a subject of study used in many areas of mathematics. The concept of metric space continues to be generalized and studied in different ways. For example, the concept of b-metric space appears as a generalization of the concept of metric space [**2**]. Later, the concept of b-metric space continued to be further generalized using auxiliary functions. In this context, the concept of extended b-metric space is introduced with the help of a θ auxiliary function [**8**]. As another example, the concept of controlled metric type space obtained with the help of the α control function can be given as a generalization of b-metric spaces [**9**]. In addition, the concepts of double controlled metric type space and double controlled metric-like space are given as new generalizations by using two noncomparable control functions such as α and μ (see, [**1**] and [**10**]).

Metric fixed-point theory can be applicable to different areas (for example, see [**5**] and [**6**]). One of them is coding theory. In [**11**], a character table was

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created using the concept of fixed point and contractive condition on the metric space, and encryption and decryption algorithms were obtained with this character table. This study is also important as it shows that fixed point theory can also be applied to coding theory.

Inspired by all the above-mentioned studies, in this article we develop a new encryption and decryption method using the concept of control function on controlled metric type space. For this purpose, we create a new character table and make use of the contractive condition and the concept of fixed point. Finally, some examples providing the resulting algorithms are given.

2. Preliminaries

We provide some fundamental ideas about metric and generalized metric spaces in this section.

Let X be a nonempty set and $d : X \times X \to [0, \infty)$ a function. If the following conditions are satisfied for all $x, y, z \in X$, then d is called a metric:

$(d1)$ $d(x, y) = 0$ if and only if $x = y$,

$(d2)$ $d(x, y) = d(y, x)$,

$(d3)$ $d(x, y) \leqslant d(x, z) + d(z, y)$.

Then the pair (X, d) is said to be a metric space.

Let X be a nonempty set and $d_A : X \times X \to [0, \infty)$ be a function defined as

$$d_A(x, y) = \begin{cases} 0 & ; \quad x = y \\ 1 & ; \quad x \neq y \end{cases},$$

for all $x, y \in X$. Then d_A is a metric on X. The metric d_A is called a discrete metric and the pair (X, d_A) is called a discrete metric space.

Various methods were employed to generalize this metric space, as seen by the concepts that follow:

Definition 2.1. [**2**] Let X be a nonempty set and $d_s : X \times X \to [0, \infty)$ be a function. If the following conditions are satisfied for all $x, y, z \in X$, then d_s is called a b-metric:

(d_s1) $d_s(x, y) = 0$ if and only if $x = y$,

(d_s2) $d_s(x, y) = d_s(y, x)$,

(d_s3) There is $s \geqslant 1$ such that

$$d_s(x, y) \leqslant s\left[d_s(x, z) + d_s(z, y)\right],$$

then the pair (X, d_s) is said to be a b-metric space.

A metric is generalized in the concept of a b-metric. In fact, if we assume $s = 1$ in Definition 2.1, then the ideas match. There are various examples of b-metrics in the literature that are not metrics.

Example 2.1. Let $X = l_p(\mathbb{R})$ with $p \in (0, 1)$ where

$$l_p(\mathbb{R}) = \left\{ \{x_n\} \subset \mathbb{R} : \sum_{n=1}^{\infty} |x_n|^p < \infty \right\}.$$

Assume that for $x = \{x_n\}$ and $y = \{y_n\}$, the function $d_s : X \times X \to [0, \infty)$ is defined by

$$d_s(x, y) = \left(\sum_{n=1}^{\infty} |x_n - y_n|^p \right)^{\frac{1}{p}}.$$

Then d_s is a b-metric with $s = 2^{\frac{1}{p}}$ (see, [**3**], [**4**] and [**7**] for more details).

Definition 2.2. [**8**] Let X be a nonempty set, $\theta : X \times X \to [1, \infty)$ and $d_\theta : X \times X \to [0, \infty)$ be two functions. If the following conditions are satisfied for all $x, y, z \in X$, then d_θ is called an extended b-metric:
($d_\theta 1$) $d_\theta(x, y) = 0$ if and only if $x = y$,
($d_\theta 2$) $d_\theta(x, y) = d_\theta(y, x)$,
($d_\theta 3$) $d_\theta(x, y) \leqslant \theta(x, y)[d_\theta(x, z) + d_\theta(z, y)]$.
Then the pair (X, d_θ) is said to be an extended b-metric space.

Example 2.2. [**8**] Let $X = \{1, 2, 3\}$ and a function $d_\theta : X \times X \to [0, \infty)$ be defined as

$$d_\theta(1, 1) = d_\theta(2, 2) = d_\theta(3, 3) = 0,$$

$$d_\theta(1, 2) = d_\theta(2, 1) = 80,$$

$$d_\theta(1, 3) = d_\theta(3, 1) = 1000$$

and

$$d_\theta(2, 3) = d_\theta(3, 2) = 600$$

for all $x, y \in X$. Then d_θ is an extended b-metric on X with the function $\theta : X \times X \to [1, \infty)$ defined as

$$\theta(x, y) = 1 + x + y,$$

for all $x, y \in X$.

Definition 2.3. [**9**] Let X be a nonempty set, $\alpha : X \times X \to [1, \infty)$ and $d_c : X \times X \to [0, \infty)$ two functions. If the following conditions are satisfied for all $x, y, z \in X$, then d_c is called a controlled metric type:
($d_c 1$) $d_c(x, y) = 0$ if and only if $x = y$,
($d_c 2$) $d_c(x, y) = d_c(y, x)$,
($d_c 3$) $d_c(x, y) \leqslant \alpha(x, z) d_c(x, z) + \alpha(z, y) d_c(z, y)$.
Then the pair (X, d_c) is said to be a controlled metric type space.

Example 2.3. [**9**] Let $X = \{1, 2, \ldots\}$ and a function $d_c : X \times X \to [0, \infty)$ be defined as

$$d_c(x, y) = \begin{cases} 0 & ; \quad x = y \\ \frac{1}{x} & ; \quad x \text{ is even and } y \text{ is odd} \\ \frac{1}{y} & ; \quad x \text{ is odd and } y \text{ is even} \\ 1 & ; \quad \text{otherwise} \end{cases},$$

for all $x, y \in X$. Then d_c is a controlled metric type on X with the function $\alpha : X \times X \to [1, \infty)$ defined as

$$\alpha(x,y) = \begin{cases} x & ; \quad x \text{ is even and } y \text{ is odd} \\ y & ; \quad x \text{ is odd and } y \text{ is even} \\ 1 & ; \quad \text{otherwise} \end{cases},$$

for all $x, y \in X$.

Definition 2.4. [**1**] Let X be a nonempty set, $\alpha, \mu : X \times X \to [1, \infty)$ non-comparable functions and $d_{cc} : X \times X \to [0, \infty)$ two functions. If the following conditions are satisfied for all $x, y, z \in X$, then d_{cc} is called a double controlled metric type:

$(d_{cc}1)$ $d_{cc}(x,y) = 0$ if and only if $x = y$,
$(d_{cc}2)$ $d_{cc}(x,y) = d_{cc}(y,x)$,
$(d_{cc}3)$ $d_{cc}(x,y) \leqslant \alpha(x,z) d_{cc}(x,z) + \mu(z,y) d_{cc}(z,y)$.
Then the pair (X, d_{cc}) is said to be a double controlled metric type space.

Example 2.4. [**1**] Let $X = [0, \infty)$ and a function $d_{cc} : X \times X \to [0, \infty)$ be defined as

$$d_{cc}(x,y) = \begin{cases} 0 & ; \quad x = y \\ \frac{1}{x} & ; \quad x \geqslant 1 \text{ and } y \in [0,1) \\ \frac{1}{y} & ; \quad y \geqslant 1 \text{ and } x \in [0,1) \\ 1 & ; \quad \text{otherwise} \end{cases},$$

for all $x, y \in X$. Then d_{cc} is a double controlled metric type on X with the functions $\alpha, \mu : X \times X \to [1, \infty)$ defined as

$$\alpha(x,y) = \begin{cases} x & ; \quad x, y \geqslant 1 \\ 1 & ; \quad \text{otherwise} \end{cases}$$

and

$$\mu(x,y) = \begin{cases} 1 & ; \quad x, y < 1 \\ \max\{x, y\} & ; \quad \text{otherwise} \end{cases},$$

for all $x, y \in X$.

Definition 2.5. [**10**] Let X be a nonempty set, $\alpha, \mu : X \times X \to [1, \infty)$ non-comparable functions and $d_{ccl} : X \times X \to [0, \infty)$ two functions. If the following conditions are satisfied for all $x, y, z \in X$, then d_{ccl} is called a double controlled metric-like:

$(d_{ccl}1)$ $d_{ccl}(x,y) = 0 \Longrightarrow x = y$,
$(d_{ccl}2)$ $d_{ccl}(x,y) = d_{ccl}(y,x)$,
$(d_{ccl}3)$ $d_{ccl}(x,y) \leqslant \alpha(x,z) d_{ccl}(x,z) + \mu(z,y) d_{ccl}(z,y)$.
Then the pair (X, d_{ccl}) is said to be a double controlled metric-like space.

EXAMPLE 2.5. [**10**] Let $X = [0, \infty)$ and a function $d_{ccl} : X \times X \to [0, \infty)$ be defined as

$$d_{ccl}(x,y) = \begin{cases} 0 & \Longrightarrow & x = y \\ \frac{1}{2} & ; & x = 0 \text{ and } y = 0 \\ \frac{1}{x} & ; & x \geqslant 1 \text{ and } y \in [0,1) \\ \frac{1}{y} & ; & y \geqslant 1 \text{ and } x \in [0,1) \\ 1 & ; & \text{otherwise} \end{cases},$$

for all $x, y \in X$. Then d_{cc} is a double controlled metric-like on X with the functions $\alpha, \mu : X \times X \to [1, \infty)$ defined as

$$\alpha(x,y) = \begin{cases} x & ; & x, y \geqslant 1 \\ 1 & ; & \text{otherwise} \end{cases}$$

and

$$\mu(x,y) = \begin{cases} 1 & ; & x, y < 1 \\ \max\{x, y\} & ; & \text{otherwise} \end{cases},$$

for all $x, y \in X$.

3. Formation of character table

Let (X, d_c) be a controlled metric type space with constant function α and the self-mapping $T_i : X \to X$ be defined as

$$T_i(x) = 1 + \frac{i-1}{i}x,\ i \in \{j : j \text{ is a prime number}\},$$

for all $x \in X$.

The characters to be used while coding are defined as follows, in order:

$$character := \alpha(x,x)\, T_i(x).$$

To create the character table, let us consider $(\mathbb{R}, d_c)$ be a controlled metric type space with the controlled metric defined as

$$d_c(x,y) = e\,|x-y|,$$

for all $x, y \in \mathbb{R}$ with the function $\alpha : \mathbb{R} \times \mathbb{R} \to [0, \infty)$ as

$$\alpha(x,y) = e,$$

for all $x, y \in \mathbb{R}$. Now, we show that the function d_c is a controlled type metric:

(d_c1) For all $x, y \in \mathbb{R}$, we have

$$d_c(x,y) = 0 \Leftrightarrow e\,|x-y| = 0 \Leftrightarrow |x-y| = 0 \Leftrightarrow x = y.$$

(d_c2) For all $x, y \in \mathbb{R}$, we have

$$d_c(x,y) = e\,|x-y| = e\,|y-x| = d_c(y,x).$$

(d_c3) For all $x, y, z \in \mathbb{R}$, we have

$$\begin{aligned} d_c(x,y) &= e|x-y| = e|x-z+z-y| \\ &\leqslant e|x-z| + e|z-y| \\ &\leqslant e^2|x-z| + e^2|z-y| \\ &= ed(x,z) + ed(z,y) \\ &= \alpha(x,z)\,d(x,z) + \alpha(z,y)\,d(z,y). \end{aligned}$$

The character table to be used is as follows:

Character Table:
$A = \alpha(x,x)\,T_2(x) = \alpha(x,x)\left(1+\frac{x}{2}\right)$
$B = \alpha(x,x)\,T_3(x) = \alpha(x,x)\left(1+\frac{2x}{3}\right)$
$C = \alpha(x,x)\,T_5(x) = \alpha(x,x)\left(1+\frac{4x}{5}\right)$
$D = \alpha(x,x)\,T_7(x) = \alpha(x,x)\left(1+\frac{6x}{7}\right)$
$E = \alpha(x,x)\,T_{11}(x) = \alpha(x,x)\left(1+\frac{10x}{11}\right)$
$F = \alpha(x,x)\,T_{13}(x) = \alpha(x,x)\left(1+\frac{12x}{13}\right)$
$G = \alpha(x,x)\,T_{17}(x) = \alpha(x,x)\left(1+\frac{16x}{17}\right)$
$H = \alpha(x,x)\,T_{19}(x) = \alpha(x,x)\left(1+\frac{18x}{19}\right)$
$I = \alpha(x,x)\,T_{23}(x) = \alpha(x,x)\left(1+\frac{22x}{23}\right)$
$J = \alpha(x,x)\,T_{29}(x) = \alpha(x,x)\left(1+\frac{28x}{29}\right)$
$K = \alpha(x,x)\,T_{31}(x) = \alpha(x,x)\left(1+\frac{30x}{31}\right)$
$L = \alpha(x,x)\,T_{37}(x) = \alpha(x,x)\left(1+\frac{36x}{37}\right)$
$M = \alpha(x,x)\,T_{41}(x) = \alpha(x,x)\left(1+\frac{40x}{41}\right)$
$N = \alpha(x,x)\,T_{43}(x) = \alpha(x,x)\left(1+\frac{42x}{43}\right)$
$O = \alpha(x,x)\,T_{47}(x) = \alpha(x,x)\left(1+\frac{46x}{47}\right)$
$P = \alpha(x,x)\,T_{53}(x) = \alpha(x,x)\left(1+\frac{52x}{53}\right)$
$Q = \alpha(x,x)\,T_{59}(x) = \alpha(x,x)\left(1+\frac{58x}{59}\right)$
$R = \alpha(x,x)\,T_{61}(x) = \alpha(x,x)\left(1+\frac{60x}{61}\right)$
$S = \alpha(x,x)\,T_{67}(x) = \alpha(x,x)\left(1+\frac{66x}{67}\right)$
$T = \alpha(x,x)\,T_{71}(x) = \alpha(x,x)\left(1+\frac{70x}{71}\right)$
$U = \alpha(x,x)\,T_{73}(x) = \alpha(x,x)\left(1+\frac{72x}{73}\right)$
$V = \alpha(x,x)\,T_{79}(x) = \alpha(x,x)\left(1+\frac{78x}{79}\right)$
$W = \alpha(x,x)\,T_{83}(x) = \alpha(x,x)\left(1+\frac{82x}{83}\right)$
$X = \alpha(x,x)\,T_{89}(x) = \alpha(x,x)\left(1+\frac{88x}{89}\right)$
$Y = \alpha(x,x)\,T_{97}(x) = \alpha(x,x)\left(1+\frac{96x}{97}\right)$
$Z = \alpha(x,x)\,T_{101}(x) = \alpha(x,x)\left(1+\frac{100x}{101}\right)$

4. Encryption and decryption algorithms

In this section, we give an encryption algorithm and a decryption algorithm using the character table defined in the previous section.

Encryption Algorithm:

1) Find the function corresponding to the character according to the character table.

2) Determine control function.

3) Create a new function by dividing the corresponding function by the control function.

4) Take the composition of these functions.

5) Find the fixed point of the composite function.

6) Substitute the fixed point in the control function and assign this value as β.

7) Multiply the fixed point by β and assign this value as Δ^*.

8) Identify fixed points corresponding to characters.

9) Create the composite number by multiplying these determined fixed points and assign this composite number as μ.

10) Send the composite number μ, the number Δ^* and control function to the channel.

Decryption Algorithm:

1) Divide the composite number into its prime factors.

2) Determine T_i functions that accept these prime factors as fixed points.

3) Multiply T_i functions by the control function.

4) Identify corresponding characters.

5) Determine the combination of functions according to the order of the characters.

6) Take the compositions of the functions found in the second step according to their corresponding character order.

7) Find the fixed points of the composite functions.

8) Substitute the fixed points into the control function.

9) Multiply the result obtained in the eighth step by the fixed point.

10) Whichever of the results obtained in the ninth step coincides with the Δ^* is the message sent.

5. Some illustrative examples

In this section, we present three examples to show the validity of encryption and decryption algorithms.

Example 5.1. Let the message to be sent is

"AB".

Encryption Algorithm:

1) Corresponding functions according to characters:

$$A = \alpha(x,x)\, T_2(x) = \alpha(x,x)\left(1+\frac{x}{2}\right)$$

and

$$B = \alpha(x,x)\, T_3(x) = \alpha(x,x)\left(1+\frac{2x}{3}\right).$$

2) Let us define the control function as follows:

$$\alpha(x,x) = e.$$

3) Let us divide the functions in the first step into the control function:

$$\frac{A}{e} = 1+\frac{x}{2} = T_2(x)$$

and

$$\frac{B}{e} = 1 + \frac{2x}{3} = T_3(x).$$

4) Let us take the composition of the functions obtained in the previous step:

$$T_2(T_3(x)) = T_2\left(1 + \frac{2x}{3}\right) = \frac{9+2x}{6}.$$

5) Let us find the fixed point of the composite function:

$$T_2(T_3(x)) = x \Longrightarrow \frac{9+2x}{6} = x \Longrightarrow x = \frac{9}{4}.$$

6) Since the control function is a constant function, β is equal to the control function:

$$\beta = e.$$

7) Let us multiply the found fixed point by β:

$$\Delta^* = \frac{9}{4}e$$

8) The fixed points corresponding to the characters are as follows, in order:

$$1 + \frac{x}{2} = x \Longrightarrow x = 2$$

and

$$1 + \frac{2x}{3} = x \Longrightarrow x = 3.$$

9) Let us calculate the composite number μ by multiplying the fixed points found in the eighth step:

$$\mu = 2 \times 3 = 6.$$

10) Let us send the following values to the channel:

$$\mu = 6,\ \Delta^* = \frac{9}{4}e \text{ and } \alpha(x,x) = e.$$

Decryption Algorithm:

1) Let us divide the composite number μ coming in the channel into its prime factors:

$$6 = 2 \times 3.$$

2) Let us determine the functions T_i that accept the prime factors obtained in the first step as fixed points:

$$T_2(x) = 1 + \frac{x}{2}$$

and

$$T_3(x) = 1 + \frac{2x}{3}.$$

3) Let us multiply the functions T_i we found by the control function α coming in the channel:

$$\alpha(x,x)\,T_2(x) = e\left(1 + \frac{x}{2}\right)$$

and

$$\alpha(x,x)\,T_3(x) = e\left(1+\frac{2x}{3}\right).$$

4) The characters corresponding to the functions obtained as a result of the operation are as follows:

$$e\left(1+\frac{x}{2}\right) \longrightarrow A$$

and

$$e\left(1+\frac{2x}{3}\right) \longrightarrow B.$$

5) There are two different combinations obtained using these two characters:

$$AB \text{ and } BA.$$

6) Let us calculate new composite functions corresponding to these combinations using the functions T_i corresponding to the characters:

$$AB \Longrightarrow T_2(T_3(x)) = \frac{9+2x}{6}$$

and

$$BA \Longrightarrow T_3(T_2(x)) = \frac{x+5}{3}.$$

7) The fixed points of the composite functions obtained in the previous step are as follows:

$$\frac{9+2x}{6} = x \Longrightarrow x = \frac{9}{4}$$

and

$$\frac{x+5}{3} = x \Longrightarrow x = \frac{5}{2}.$$

8) Since the control function is a constant function, the result is the same for two fixed points:

$$\alpha\left(\frac{9}{4},\frac{9}{4}\right) = \alpha\left(\frac{5}{2},\frac{5}{2}\right) = e.$$

9) Let us multiply the result obtained in the previous step by the fixed points:

$$\frac{9}{4}e \text{ and } \frac{5}{2}e.$$

10) The value that coincides with the number Δ^* coming in the channel is the message we are looking for:

$$\Delta^* = \frac{9}{4}e \Longrightarrow \text{message : “AB”}.$$

EXAMPLE 5.2. Let the message to be sent is

“BUSE”.

Encryption Algorithm:

1) Corresponding functions according to characters:

$$B = \alpha(x,x)\,T_3(x) = \alpha(x,x)\left(1+\frac{2x}{3}\right),$$

$$U=\alpha\left(x,x\right)T_{73}\left(x\right)=\alpha\left(x,x\right)\left(1+\frac{72x}{73}\right),$$

$$S=\alpha\left(x,x\right)T_{67}\left(x\right)=\alpha\left(x,x\right)\left(1+\frac{66x}{67}\right)$$

and

$$E=\alpha\left(x,x\right)T_{11}\left(x\right)=\alpha\left(x,x\right)\left(1+\frac{10x}{11}\right).$$

2) Let us define the control function as follows:

$$\alpha\left(x,x\right)=e.$$

3) Let us divide the functions in the first step into the control function:

$$\frac{B}{e}=1+\frac{2x}{3}=T_{3}\left(x\right),$$

$$\frac{U}{e}=1+\frac{72x}{73}=T_{73}\left(x\right),$$

$$\frac{S}{e}=1+\frac{66x}{67}=T_{67}\left(x\right)$$

and

$$\frac{E}{e}=1+\frac{10x}{11}=T_{11}\left(x\right).$$

4) Let us take the composition of the functions obtained in the previous step:

$$\begin{aligned}T_{3}\left(T_{73}\left(T_{67}\left(T_{11}\left(x\right)\right)\right)\right) &= T_{3}\left(T_{73}\left(T_{67}\left(1+\frac{10x}{11}\right)\right)\right)\\ &= T_{3}\left(T_{73}\left(\frac{660x+1463}{737}\right)\right)\\ &= T_{3}\left(\frac{47520x+159137}{53801}\right)\\ &= \frac{95040x+479677}{161403}.\end{aligned}$$

5) Let us find the fixed point of the composite function:

$$\begin{aligned}T_{3}\left(T_{73}\left(T_{67}\left(T_{11}\left(x\right)\right)\right)\right) &= x\Longrightarrow\frac{95040x+479677}{161403}=x\\ &\Longrightarrow x=\frac{479677}{66363}.\end{aligned}$$

6) Since the control function is a constant function, β is equal to the control function:

$$\beta=e.$$

7) Let us multiply the found fixed point by β:

$$\Delta^{*}=\frac{479677}{66363}e$$

8) The fixed points corresponding to the characters are as follows, in order:

$$1+\frac{2x}{3}=x\Longrightarrow x=3,$$

$$1+\frac{72x}{73}=x\Longrightarrow x=73,$$

$$1+\frac{66x}{67}=x\Longrightarrow x=67$$

and

$$1+\frac{10x}{11}=x\Longrightarrow x=11.$$

9) Let us calculate the composite number μ by multiplying the fixed points found in the eighth step:

$$\mu=3\times 73\times 67\times 11=161403.$$

10) Let us send the following values to the channel:

$$\mu=161403,\ \Delta^{*}=\frac{479677}{66363}e \text{ and } \alpha\left(x,x\right)=e.$$

Decryption Algorithm:

1) Let us divide the composite number μ coming in the channel into its prime factors:

$$161403=3\times 11\times 67\times 73.$$

2) Let us determine the functions T_i that accept the prime factors obtained in the first step as fixed points:

$$T_{3}\left(x\right)=1+\frac{2x}{3},$$

$$T_{11}\left(x\right)=1+\frac{10x}{11},$$

$$T_{67}\left(x\right)=1+\frac{66x}{67}$$

and

$$T_{73}\left(x\right)=1+\frac{72x}{73}.$$

3) Let us multiply the functions T_i we found by the control function α coming in the channel:

$$\alpha\left(x,x\right)T_{3}\left(x\right)=e\left(1+\frac{2x}{3}\right),$$

$$\alpha\left(x,x\right)T_{11}\left(x\right)=e\left(1+\frac{10x}{11}\right),$$

$$\alpha\left(x,x\right)T_{67}\left(x\right)=e\left(1+\frac{66x}{67}\right)$$

and

$$\alpha\left(x,x\right)T_{73}\left(x\right)=e\left(1+\frac{72x}{73}\right).$$

4) The characters corresponding to the functions obtained as a result of the operation are as follows:

$$e\left(1+\frac{2x}{3}\right)\longrightarrow B,$$

$$e\left(1+\frac{10x}{11}\right)\longrightarrow E,$$

$$e\left(1+\frac{66x}{67}\right)\longrightarrow S$$

and

$$e\left(1+\frac{72x}{73}\right)\longrightarrow U.$$

5) There are $4! = 24$ different combinations obtained using these two characters:

$BUSE$	$BSEU$	$BEUS$	$BUES$	$BESU$	$BSUE$
$USEB$	$UESB$	$USBE$	$UEBS$	$UBES$	$UBSE$
$SEBU$	$SEUB$	$SBUE$	$SBEU$	$SUBE$	$SUEB$
$ESUB$	$EUBS$	$EUSB$	$EBSU$	$EBUS$	$ESBU$

.

6) Let us calculate new composite functions corresponding to these combinations using the functions T_i corresponding to the characters:

$$BUSE \Longrightarrow T_3\left(T_{73}\left(T_{67}\left(T_{11}\left(x\right)\right)\right)\right)=\frac{95040x+479677}{161403},$$

$$BSEU \Longrightarrow T_3\left(T_{67}\left(T_{11}\left(T_{73}\left(x\right)\right)\right)\right)=\frac{95040x+471361}{161403},$$

$$BEUS \Longrightarrow T_3\left(T_{11}\left(T_{73}\left(T_{67}\left(x\right)\right)\right)\right)=\frac{95040x+463305}{161403},$$

$$BUES \Longrightarrow T_3\left(T_{73}\left(T_{11}\left(T_{67}\left(x\right)\right)\right)\right)=\frac{95040x+471613}{161403},$$

$$BESU \Longrightarrow T_3\left(T_{11}\left(T_{67}\left(T_{73}\left(x\right)\right)\right)\right)=\frac{95040x+463185}{161403},$$

$$BSUE \Longrightarrow T_3\left(T_{67}\left(T_{73}\left(T_{11}\left(x\right)\right)\right)\right)=\frac{95040x+479545}{161403},$$

$$USEB \Longrightarrow T_{73}\left(T_{67}\left(T_{11}\left(T_3\left(x\right)\right)\right)\right)=\frac{95040x+619971}{161403},$$

$$UESB \Longrightarrow T_{73}\left(T_{11}\left(T_{67}\left(T_3\left(x\right)\right)\right)\right)=\frac{95040x+607875}{161403},$$

$$USBE \Longrightarrow T_{73}\left(T_{67}\left(T_3\left(T_{11}\left(x\right)\right)\right)\right)=\frac{95040x+581955}{161403},$$

$$UEBS \Longrightarrow T_{73}\left(T_{11}\left(T_3\left(T_{67}\left(x\right)\right)\right)\right)=\frac{95040x+561795}{161403},$$

$$UBES \Longrightarrow T_{73}\left(T_3\left(T_{11}\left(T_{67}\left(x\right)\right)\right)\right)=\frac{95040x+523203}{161403},$$

$$UBSE \Longrightarrow T_{73}\left(T_3\left(T_{67}\left(T_{11}\left(x\right)\right)\right)\right)=\frac{95040x+531267}{161403},$$

$$SEBU \Longrightarrow T_{67}\left(T_{11}\left(T_3\left(T_{73}\left(x\right)\right)\right)\right)=\frac{95040x+561297}{161403},$$

$$SEUB \Longrightarrow T_{67}\left(T_{11}\left(T_{73}\left(T_{3}\left(x\right)\right)\right)\right) = \frac{95040x + 607497}{161403},$$
$$SBUE \Longrightarrow T_{67}\left(T_{3}\left(T_{73}\left(T_{11}\left(x\right)\right)\right)\right) = \frac{95040x + 530937}{161403},$$
$$SBEU \Longrightarrow T_{67}\left(T_{3}\left(T_{11}\left(T_{73}\left(x\right)\right)\right)\right) = \frac{95040x + 522753}{161403},$$
$$SUBE \Longrightarrow T_{67}\left(T_{73}\left(T_{3}\left(T_{11}\left(x\right)\right)\right)\right) = \frac{95040x + 581757}{161403},$$
$$SUEB \Longrightarrow T_{67}\left(T_{73}\left(T_{11}\left(T_{3}\left(x\right)\right)\right)\right) = \frac{95040x + 623931}{161403},$$
$$ESUB \Longrightarrow T_{11}\left(T_{67}\left(T_{73}\left(T_{3}\left(x\right)\right)\right)\right) = \frac{95040x + 589833}{161403},$$
$$EUBS \Longrightarrow T_{11}\left(T_{73}\left(T_{3}\left(T_{67}\left(x\right)\right)\right)\right) = \frac{95040x + 889173}{161403},$$
$$EUSB \Longrightarrow T_{11}\left(T_{73}\left(T_{67}\left(T_{3}\left(x\right)\right)\right)\right) = \frac{95040x + 595413}{161403},$$
$$EBSU \Longrightarrow T_{11}\left(T_{3}\left(T_{67}\left(T_{73}\left(x\right)\right)\right)\right) = \frac{95040x + 502313}{161403},$$
$$EBUS \Longrightarrow T_{11}\left(T_{3}\left(T_{73}\left(T_{67}\left(x\right)\right)\right)\right) = \frac{95040x + 502433}{161403}$$

and

$$ESBU \Longrightarrow T_{11}\left(T_{67}\left(T_{3}\left(T_{73}\left(x\right)\right)\right)\right) = \frac{95040x + 549033}{161403}.$$

7) The fixed points of the composite functions obtained in the previous step are as follows:

$$\frac{95040x + 479677}{161403} = x \Longrightarrow x = \frac{479677}{66363},$$
$$\frac{95040x + 471361}{161403} = x \Longrightarrow x = \frac{471361}{66363},$$
$$\frac{95040x + 463305}{161403} = x \Longrightarrow x = \frac{463305}{66363},$$
$$\frac{95040x + 471613}{161403} = x \Longrightarrow x = \frac{471613}{66363},$$
$$\frac{95040x + 463185}{161403} = x \Longrightarrow x = \frac{463185}{66363},$$
$$\frac{95040x + 479545}{161403} = x \Longrightarrow x = \frac{479545}{66363},$$
$$\frac{95040x + 619971}{161403} = x \Longrightarrow x = \frac{619971}{66363},$$
$$\frac{95040x + 607875}{161403} = x \Longrightarrow x = \frac{607875}{66363},$$
$$\frac{95040x + 581955}{161403} = x \Longrightarrow x = \frac{581955}{66363},$$
$$\frac{95040x + 561795}{161403} = x \Longrightarrow x = \frac{561795}{66363},$$
$$\frac{95040x + 523203}{161403} = x \Longrightarrow x = \frac{523203}{66363},$$

$$\frac{95040x + 531267}{161403} = x \Longrightarrow x = \frac{531267}{66363},$$
$$\frac{95040x + 561297}{161403} = x \Longrightarrow x = \frac{561297}{66363},$$
$$\frac{95040x + 607497}{161403} = x \Longrightarrow x = \frac{607497}{66363},$$
$$\frac{95040x + 530937}{161403} = x \Longrightarrow x = \frac{530937}{66363},$$
$$\frac{95040x + 522753}{161403} = x \Longrightarrow x = \frac{522753}{66363},$$
$$\frac{95040x + 581757}{161403} = x \Longrightarrow x = \frac{581757}{66363},$$
$$\frac{95040x + 623931}{161403} = x \Longrightarrow x = \frac{623931}{66363},$$
$$\frac{95040x + 589833}{161403} = x \Longrightarrow x = \frac{589833}{66363},$$
$$\frac{95040x + 889173}{161403} = x \Longrightarrow x = \frac{889173}{66363},$$
$$\frac{95040x + 595413}{161403} = x \Longrightarrow x = \frac{595413}{66363},$$
$$\frac{95040x + 502313}{161403} = x \Longrightarrow x = \frac{502313}{66363},$$
$$\frac{95040x + 502433}{161403} = x \Longrightarrow x = \frac{502433}{66363}$$

and

$$\frac{95040x + 549033}{161403} = x \Longrightarrow x = \frac{549033}{66363}.$$

8) Since the control function is a constant function, the result is the same for 24 fixed points:

$$\alpha(x, x) = e,$$

where x is a fixed point obtained in the previous step.

9) Let us multiply the result obtained in the previous step by the fixed points:

$\frac{479677}{66363}e$	$\frac{471361}{66363}e$	$\frac{463305}{66363}e$	$\frac{471613}{66363}e$	$\frac{463185}{66363}e$	$\frac{479545}{66363}e$
$\frac{619971}{66363}e$	$\frac{607875}{66363}e$	$\frac{581955}{66363}e$	$\frac{561795}{66363}e$	$\frac{523203}{66363}e$	$\frac{531267}{66363}e$
$\frac{561297}{66363}e$	$\frac{607497}{66363}e$	$\frac{530937}{66363}e$	$\frac{522753}{66363}e$	$\frac{581757}{66363}e$	$\frac{623931}{66363}e$
$\frac{589833}{66363}e$	$\frac{889173}{66363}e$	$\frac{595413}{66363}e$	$\frac{502313}{66363}e$	$\frac{502433}{66363}e$	$\frac{549033}{66363}e$

10) The value that coincides with the number Δ^* coming in the channel is the message we are looking for:

$$\Delta^* = \frac{479677}{66363}e \Longrightarrow \text{message : “BUSE”}.$$

EXAMPLE 5.3. Let the message to be sent is

"TAS".

Encryption Algorithm:

1) Corresponding functions according to characters:

$$T = \alpha(x, x) T_{71}(x) = \alpha(x, x)\left(1 + \frac{70x}{71}\right),$$

$$A = \alpha(x, x) T_2(x) = \alpha(x, x)\left(1 + \frac{x}{2}\right)$$

and

$$S = \alpha(x, x) T_{67}(x) = \alpha(x, x)\left(1 + \frac{66x}{67}\right).$$

2) Let us define the control function as follows:

$$\alpha(x, x) = x^2 + 1.$$

3) Let us divide the functions in the first step into the control function:

$$\frac{T}{x^2 + 1} = 1 + \frac{70x}{71} = T_{71}(x),$$

$$\frac{A}{x^2 + 1} = 1 + \frac{x}{2} = T_2(x)$$

and

$$\frac{S}{x^2 + 1} = 1 + \frac{66x}{67} = T_{67}(x).$$

4) Let us take the composition of the functions obtained in the previous step:

$$\begin{aligned} T_{71}(T_2(T_{67}(x))) &= T_{71}\left(T_2\left(1 + \frac{66x}{67}\right)\right) \\ &= T_{71}\left(\frac{66x + 201}{134}\right) \\ &= \frac{4620x + 23584}{9514}. \end{aligned}$$

5) Let us find the fixed point of the composite function:

$$\begin{aligned} T_{71}(T_2(T_{67}(x))) &= x \Longrightarrow \frac{4620x + 23584}{9514} = x \\ &\Longrightarrow x = \frac{23584}{4894}. \end{aligned}$$

6) Since the control function is $\alpha(x, x) = x^2 + 1$, then we have

$$\beta = \frac{580156292}{23951236}.$$

7) Let us multiply the found fixed point by β:

$$\Delta^* = \frac{13682405990528}{117217348984}.$$

8) The fixed points corresponding to the characters are as follows, in order:

$$1+\frac{70x}{71}=x\Longrightarrow x=71,$$
$$1+\frac{x}{2}=x\Longrightarrow x=2$$

and

$$1+\frac{66x}{67}=x\Longrightarrow x=67.$$

9) Let us calculate the composite number μ by multiplying the fixed points found in the eighth step:

$$\mu=71\times 2\times 67=9514.$$

10) Let us send the following values to the channel:

$$\mu=9514,\ \Delta^{*}=\frac{13682405990528}{117217348984}\text{ and }\alpha\left(x,x\right)=x^{2}+1.$$

Decryption Algorithm:

1) Let us divide the composite number μ coming in the channel into its prime factors:

$$9514=2\times 67\times 71.$$

2) Let us determine the functions T_i that accept the prime factors obtained in the first step as fixed points:

$$T_{2}\left(x\right)=1+\frac{x}{2},$$
$$T_{67}\left(x\right)=1+\frac{66x}{67}$$

and

$$T_{71}\left(x\right)=1+\frac{70x}{71}.$$

3) Let us multiply the functions T_i we found by the control function α coming in the channel:

$$\alpha\left(x,x\right)T_{2}\left(x\right)=\left(x^{2}+1\right)\left(1+\frac{x}{2}\right),$$
$$\alpha\left(x,x\right)T_{67}\left(x\right)=\left(x^{2}+1\right)\left(1+\frac{66x}{67}\right)$$

and

$$\alpha\left(x,x\right)T_{71}\left(x\right)=\left(x^{2}+1\right)\left(1+\frac{70x}{71}\right).$$

4) The characters corresponding to the functions obtained as a result of the operation are as follows:

$$\left(x^{2}+1\right)\left(1+\frac{x}{2}\right)\longrightarrow A,$$
$$\left(x^{2}+1\right)\left(1+\frac{66x}{67}\right)\longrightarrow S$$

and

$$\left(x^{2}+1\right)\left(1+\frac{70x}{71}\right)\longrightarrow T.$$

5) There are $3! = 6$ different combinations obtained using these two characters:

TAS	TSA	ATS	AST	STA	SAT

.

6) Let us calculate new composite functions corresponding to these combinations using the functions T_i corresponding to the characters:

$$TAS \Longrightarrow T_{71}\left(T_2\left(T_{67}\left(x\right)\right)\right) = \frac{4620x + 23584}{9514},$$

$$TSA \Longrightarrow T_{71}\left(T_{67}\left(T_2\left(x\right)\right)\right) = \frac{4620x + 28134}{9514},$$

$$ATS \Longrightarrow T_2\left(T_{71}\left(T_{67}\left(x\right)\right)\right) = \frac{4620x + 18961}{9514},$$

$$AST \Longrightarrow T_2\left(T_{67}\left(T_{71}\left(x\right)\right)\right) = \frac{4620x + 18957}{9514},$$

$$STA \Longrightarrow T_{67}\left(T_{71}\left(T_2\left(x\right)\right)\right) = \frac{4620x + 28126}{9514}$$

and

$$SAT \Longrightarrow T_{67}\left(T_2\left(T_{71}\left(x\right)\right)\right) = \frac{4620x + 23572}{9514}.$$

7) The fixed points of the composite functions obtained in the previous step are as follows:

$$\frac{4620x + 23584}{9514} = x \Longrightarrow x = \frac{23584}{4894},$$

$$\frac{4620x + 28134}{9514} = x \Longrightarrow x = \frac{28134}{5254},$$

$$\frac{4620x + 18961}{9514} = x \Longrightarrow x = \frac{18961}{4894},$$

$$\frac{4620x + 18957}{9514} = x \Longrightarrow x = \frac{18957}{4894},$$

$$\frac{4620x + 28126}{9514} = x \Longrightarrow x = \frac{28126}{4894}$$

and

$$\frac{4620x + 23572}{9514} = x \Longrightarrow x = \frac{23572}{4894}.$$

8) Let x be a fixed point obtained in the previous step. Then we have the followings:

$$x = \frac{23584}{4894} \Longrightarrow \alpha\left(x, x\right) = \frac{580156292}{23951236},$$

$$x = \frac{28134}{5254} \Longrightarrow \alpha\left(x, x\right) = \frac{819126472}{27604516},$$

$$x = \frac{18961}{4894} \Longrightarrow \alpha\left(x, x\right) = \frac{383470757}{23951236},$$

$$x = \frac{18957}{4894} \Longrightarrow \alpha\left(x, x\right) = \frac{383319085}{23951236},$$

$$x = \frac{28126}{4894} \Longrightarrow \alpha\left(x, x\right) = \frac{815023112}{23951236}$$

and

$$x = \frac{23572}{4894} \Longrightarrow \alpha(x, x) = \frac{579590420}{23951236}.$$

9) Let us multiply the result obtained in the previous step by the fixed points:

$$\frac{13682405990528}{117217348984},$$
$$\frac{25858704163248}{145034127064},$$
$$\frac{7270989023477}{117217348984},$$
$$\frac{22923340048112}{117217348984},$$
$$\frac{7266579894345}{117217348984}$$

and

$$\frac{13662105380240}{117217348984}.$$

10) The value that coincides with the number Δ^* coming in the channel is the message we are looking for:

$$\Delta^* = \frac{13682405990528}{117217348984} \Longrightarrow \text{message : "TAS"}.$$

6. Conclusion

In this study, a new encryption and decryption algorithm on controlled metric type space is given. The main purpose of this study is to show that the generalization of the metric space approach used in fixed point theory is also applicable in coding theory. As an open problem, the same problem can be left to work on double controlled metric type spaces and double controlled metric-like spaces. In this sense, this article gains importance as it sheds light on new research topics.

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